AN EXPERIMENTAL INVESTIGATION OF SLOW INTEGER TUNE CROSSING IN THE EMMA NON-SCALING FFAG*

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Abstract

Results are presented from a slow integer tune crossing experiment performed in the EMMA ns-FFAG accelerator. Under nominal conditions EMMA accelerates an electron beam from 10 to 20 MeV rapidly in 5 to 10 turns in a novel serpentine channel causing several transverse integer tunes to be crossed. During this rapid acceleration it has been shown that the amplitude of the beam does not grow if the potential of non-scaling FFAGs were to be realised in such fields as high-current proton acceleration then tune space would be crossed slower with acceleration in an RF bucket. The crossing speed in a non-scaling FFAG is in a previously unstudied intermediate region and hence conventional crossing theory may not apply. It was proposed to observe the effects on beam oscillation amplitude when a beam crosses integer tunes by the variation of tune with momentum over a range of crossing speeds derived from different acceleration rates. This method can be realised by synchrotron acceleration inside a stable RF bucket. Betatron amplitude growth and beam loss as a function of turn are explored when crossing integer tunes.

INTRODUCTION

A fixed-field, alternating-gradient (FFAG) accelerator may be used to rapidly accelerate heavy particles without the need to cycle the dipole magnet fields [1, 2]. FFAGs are therefore advantageous for accelerating unstable particles such as muons used for neutrino beam generation [3], and can overcome the relativistic limitations inherent in cyclotrons to deliver high-current beams at 1 GeV for either direct neutron production or in the related application of accelerator-driven reactor systems (ADS). Proof-of-principle proton FFAG demonstrations in recent years [8, 9] have been made possible by the development of radiofrequency accelerating systems with rapidly varying frequency, and have shown that a high repetition rate above 1 kHz is in principle possible.

So-called scaling FFAGs are design so that the average bending field varies non-linearly with radius as \(< B > \sim r^k\): this allows the betatron tunes to remain constant during acceleration so that disruptive betatron resonances are not crossed, but as with a cyclotron a large accelerating range requires large-aperture magnets. Linear non-scaling FFAGs (nsFFAGs) are one way of breaking the scaling condition, such that the average magnetic field variation is linear with radius, \(< B > \sim kr\). For a given acceleration range a linear nsFFAG can be designed to need smaller magnets than the equivalent scaling lattice, but resonances are now typically crossed.

An injected bunch may tolerate crossing an integer tune if the crossing rate is fast enough, as predicted by theoretical studies [16, 17]. The EMMA nsFFAG was constructed as an electron model to test this idea [10], and has already demonstrated fast integer tune crossing over its momentum range of 10.5 MeV/c to 20.5 MeV/c, during which the betatron tunes reduce from \(\sim 10/\text{turn}\) to \(\sim 5/\text{turn}\) [11]. Recent work has concentrated on refining the comparison between simulation models of the accelerator and experiment [12].

A key measurement is to determine how slowly integer tunes may be successfully crossed, as this will guide how nsFFAGs may be applied to proton acceleration [4, 5, 6, 7].

FAST RESONANCE CROSSING

We use Baartman’s approximation for fast integer tune crossing [13, 14, 15] where the coherent amplitude \(\Delta A\) of a bunch around its closed orbit is related to the tune change per turn \(Q' = dQ/dn\) as

\[
\Delta A = \frac{\pi R B_n}{\sqrt{Q'}} \left( \frac{B_n}{Q} \right)
\]

where \(R\) is the average magnetic bending field strength, \(\bar{B}\) is the average machine radius, and \(B_n\) is the \(n\)th harmonic content of the error field. The dominant error in the horizontal plane is thought to be stray field from the injection septum [12]. Simulations have already been performed to examine the likely influence of resonance crossing in proton FFAGs [16, 17], but with EMMA we have measured the amplitude growth experimentally and compared it with a PyZgoubi tracking simulation [18, 19].

INTEGER CROSSING IN EMMA

Integer tune crossing at different speeds may be achieved by injecting a bunch into a radiofrequency bucket and allowing it to perform synchrotron oscillations, the lattice conditions and injection energy arranged such that a varying number of either horizontal or vertical transverse integer tunes are crossed. The crossing rate depends on both the accelerating voltage, and on the injected phase of the bunch with respect to the stable fixed point (SFP). Regions of tune space were crossed at acceleration rates of between 0.2 and 2.0 MV per turn. A 12.5MeV/c bunch was injected into EMMA and the technique of equivalent momentum [11] was used to mimic other injected momenta. Initial longitudinal phase of the injected bunch was varied.
from $-\pi$ to $\pi$ by changing the RF cavity phase. 42 BPMs each at equivalent positions in each cell (and therefore with the same nominal Twiss values) were used to measure the closed-orbit distortion (COD) as function of energy (by averaging over a small number of turns or by operating at fixed energy) [11] and to determine the coherent oscillation amplitude with respect to this closed orbit. The phase and thus time of flight of the bunch was determined turn-by-turn by comparing the signal from one BPM with the RF clock frequency. Assuming the acceleration rate is slow enough, the mean COD can be determined by averaging over a number of BPMs (21 in our case), from which the bunch momentum can be reconstructed; the betatron tunes may derived from the BPM signals in a similar manner [11], with the total charge proportional to the BPMs’ sum signal. Coherent amplitude is taken to be the standard deviation around the mean orbit at each BPM.

**EXPERIMENTAL DATA**

Figs. 1 and 2 show how different crossing speeds can give quite different behaviour for the same accelerating voltage. For motion close to the separatrix (Fig. 1) where the crossing speed is relatively slow, the beam experiences large amplitude growth of several mm and eventual complete beam loss when crossing a vertical integer tune. Nearer to the SFP (Fig. 2) the crossing rate is higher, and no net amplitude growth is evident. The behaviour at accelerating voltages of 0.5 MV/turn and 1.0 MV/turn was observed to be similar.

![Figure 1: Amplitude growth and longitudinal phase space for a bunch accelerated at 0.2 MV/turn and crossing speeds of $Q' \approx 0.07$. Red/blue indicate horizontal/vertical integer tunes. Similar behaviour is seen at other accelerating voltages.](image1)

![Figure 2: Similar to Fig. 1 but with injection near to the SFP, crossing speed $Q' \approx 0.09$. No net amplitude growth is evident.](image2)

**SINGLE PARTICLE STUDY WITH PYZGOUBI**

PyZgoubi [18, 19] was used to calculate the amplitude growth for a single particle in a lattice with a single vertical dipole field error of 0.1 mT·m; we expect only an amplitude growth in the horizontal plane [20]. BPM data equivalent to the experimental measurements is taken from the simulation, which simulated the behaviour a particle initially placed on the closed orbit but with varying phase. Figs. 3 and 4 shows that similar behaviour to experimental measurements can be obtained. Fig. 5 shows a linear relationship of amplitude with $1/\sqrt{Q'}$ as expected, over a range of accelerating voltages from 0.2 and 1.0 MV/turn and crossing rates $Q'$ between 0.05 and 1.5.

**DISCUSSION**

Preliminary results show that broadly similar behaviour can be obtained in simulation to that seen experimentally. Simulation data shows growth behaviour that follows the Baartman approximation. We expect experimental behaviour to follow this, but insufficient data is available to date to draw a firm conclusion. Further data collection is planned during the latter half of 2012.

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Figure 3: Acceleration of a single particle in a PyZgoubi model of EMMA, the particle injected near to the separatrix such that the crossing rate is small. Amplitude growth is derived from the standard deviation of oscillations before and after times corresponding to the width of the horizontal integer tune (shown in red, vertical integer tunes shown in blue). Acceleration voltage was 0.2 MV/turn, crossing rate $Q' = 0.07$.

Figure 4: Acceleration of a single particle in a PyZgoubi model of EMMA, the particle injected close to the SFP such that the crossing rate is larger than in Fig. 3. Acceleration voltage was 0.2 MV/turn, crossing rate $Q' = 0.09$.

REFERENCES

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