STUDIES OF ERHIC COHERENT INSTABILITIES *

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Abstract
In the presence of an effective coherent electron cooling, the rms ion bunch length in eRHIC will be kept at 8.3 cm for 250 GeV protons, which is much shorter than the current RHIC 45 cm rms bunch length. Together with the increased bunch intensity and total bunch number, coherent instabilities could be a potential limitation for achieving desired machine performance. In this study, we use the tracking code TRANFT to find thresholds and growth rates for single bunch and coupled bunch instabilities with linear chromaticity and amplitude dependent tune shift taken into account [1]. Based on the simulation results, requirements of machine parameters such as rf voltage, linear chromaticity, and tune dependence of betatron amplitude are specified to suppress these instabilities.

INTRODUCTION
In order to achieve \(10^{34} \text{cm}^{-2}\text{s}^{-1}\) level luminosity in eRHIC, the proposed electron ion collider at Brookhaven National Laboratory, short bunch length is desired to avoid substantial luminosity reduction due to hour glass effects. On the other hand, for given transverse emittance and bunch intensity, shorter bunch length leads to stronger nonlinear space charge force and beam-beam interaction, which can put circulating protons on machine resonances and hence shorten beam lifetime. For eRHIC design parameters, the space charge tune spread is conservatively chosen to be 0.035, which give 8.3 cm of rms bunch length for 250 GeV proton beam. In addition, eRHIC baseline design plan to have 166 bunches with \(2 \times 10^{11}\) protons per bunch, which are 50% increases in the bunch number and 30% increases in bunch intensity from that of current RHIC operations. With the reduced bunch length and increased bunch number, collective beam instabilities due to machine impedance could potentially limit the machine performance and hence need to be investigated.

In this work, we use a FFT-based multi-purpose coherent instability simulation code, TRANFT, to study longitudinal and transverse instabilities for the baseline parameters of eRHIC proton beam. The energy spread threshold for longitudinal microwave instabilities found by the code agree with Keil-Schnell criteria. With linear beam transport and +2 units of linear chromaticity, transverse coupled bunch instability is observed from simulation. While it is possible to suppress the instability by increasing linear chromaticity, simulation shows that introducing amplitude dependent tune shift is more efficient in stabilizing beam while keeping tune spread relatively small.

LONGITUDINAL MICROWAVE INSTABILITY

Longitudinal RHIC impedance used in the simulation include previously measured 3 ohm inductive broad band impedance and the resistive wall impedance[2]. The broad-band impedance is modelled as a resonator with \(Q = 2\) and \(f_r = 2\text{GHz}\), and low frequency formula,

\[
Z_{RW}(\omega) = \left[1 - isgn(\omega)\right]\frac{Z_0\delta_{skin}}{2b}\omega_0,
\]

is applied for resistive wall impedance, where \(Z_0\) is the impedance of free space, \(\omega_0\) is the revolution angular frequency, \(b\) and \(\delta_{skin}\) are the radius and skin depth of the beam pipe respectively. The longitudinal impedance \(Z_t/p\) and initial proton bunch spectrum are plotted in fig.1.

![Figure 1. Longitudinal impedance, \(Z_t/p\), used in the simulation. Red dot curve is the fourier transformation of initial proton beam line charge density with 8.3 cm in arbitrary unit.](image)

We assume 197 MHz rf system and use \(3 \times 10^4\) macro-particles to represent the proton bunch. As shown in fig. 2, the minimal rf voltage to keep longitudinal stability is around 4.5 MV, which lead to a rms energy spread of \(2.2 \times 10^{-4}\). Taking \(|Z_t/p| = 5\text{ohm}\) and \(I_A = 45\text{A}\) as obtained from the rms bunch length, the Keil-Schnell criteria derived for a coasting beam yields

\[
\sigma_{p,th} = \sqrt{\frac{|Z_0/p|\mu_{peak}Z_e}{(2\ln 2)m_c^2\gamma_0\beta_0^2|\eta|}} = 2.274 \times 10^{-4},
\]

which agree with the simulation results.
**TRANVERSE COUPLED BUNCH INSTABILITY**

With the help of an effective transverse coherent electron cooling, the normalized rms emittance of proton beam in eRHIC will be reduced from $2\pi \cdot \mu m$ to 0.18 $\pi \cdot \mu m$. Consequently, transverse space charge impedance is increased by an order of magnitude and dominates the total reactive impedance. Fig. 3 shows the transverse impedance used in the simulation, which include the contribution from space charge, bellows, resistive wall, bmps and abort kicker[3, 4]. The bunch repetition rate of eRHIC will be 14.1MHz, corresponding to 180 bunches pattern in RHIC ring. We neglect the abort gap in the simulation and consider 180 evenly distributed bunches. For rigid coupled bunch mode, proton beam with abort gap should be at least as stable as that without the abort gap. TRANFT accounts for multi-bunch effects by assuming all bunches are identical and consider a specific multi-bunch mode[5]. For cold bunches, the coherent tune shift is given by

$$\Delta Q = \frac{N e^2 T_0}{8 \pi^2 m_c e Q_s} \sum_{i=1}^{N} \int W_i \left( \frac{k T_0}{M} \right) \exp \left( i \left( \psi_0 - 2 \pi s \right) k \right) dt,$$

where $N$ is the number of protons in a bunch, $M$ is the total number of bunches in the ring, $Q_s$ is the betatron tune, $\psi_0$ is the betatron phase advance in one turn and $s$ is the multi-bunch mode number. Eq. (2) suggests that the most unstable mode will be the one with mode number closest to the betatron tune. Taking the typical RHIC betatron tune of 28.695, we consider the multi-bunch mode with $s = 29$. The rise time for cold proton bunches without Landau damping can be calculated from eq. (2) as

$$t_{rise} = \frac{T_0}{2 \pi \text{Im} (\Delta Q)}.$$

For eRHIC parameters, the growth time calculated from eq. (3) is 2000 turns. The transverse coherence is monitored by the quantity,

$$\text{Coherence} = \frac{\int I(t)[\langle x(t) \rangle]^2 + \langle p(t) \rangle^2] dt}{\int I(t) dt},$$

where $I(t)$ is the instantaneous current, $\langle x(t) \rangle$ is the average displacement at longitudinal location $t$ and $\langle p(t) \rangle$ is the average transverse angle multiplied the average beta function. As shown in fig. 4(a), beam is stable with multi-bunch kick turned off. When the multi-bunch kick is turned on, beam transverse motion becomes unstable. For 2 units of chromaticity, the coherence grows exponentially with a rise time of 1200 turns. While increasing chromaticity does help to reduce the instability growth, it requires more than 25 units to suppress the instability completely, which introduces of $10^{-3}$ rms betatron tune spread with the rms energy spread of $2.2 \times 10^{-4}$. 

Figure 2. Simulation results of longitudinal microwave instability. (a) Evolution of proton bunch rms energy spread for 3 MV (red), 4 MV(green) and 4.5 MV (blue) rf voltage; (b) Proton bunch instantaneous current profile after 5000 turns for 3MV (red) and 4.5 MV (blue) rf voltage.

Figure 3. Transverse impedance used in the simulation. Red dot curve is the spectrum of beam. The imaginary part of the impedance is scaled down by a factor of 100 for better visibility.

\[ \Delta Q = \frac{N e^2 T_0}{8 \pi^2 m_c e Q_s} \sum_{i=1}^{N} \int W_i \left( \frac{k T_0}{M} \right) \exp \left( i \left( \psi_0 - 2 \pi s \right) k \right) dt, \]
Another possible mechanism for increasing Landau damping is by introducing amplitude dependant tune shift. In the simulation, we consider linear tune dependence of betatron amplitude which can be introduced by octupole field. The one turn betatron phase advance for protons with betatron amplitude $J$ reads

$$\psi = \psi_0 + \xi_x \delta + J \cdot \frac{d\psi}{dJ},$$

where $\xi_x$ is the linear chromaticity and $d\psi/dJ$ is determined by the strength of the nonlinear field. As shown in fig. 5, the minimal required $d\psi/dJ$ is $4 \times 10^5m^2$, which introduces a rms tune spread of $1.3 \times 10^{-3}$.

**REFERENCES**


**SUMMARY**

According to the simulation, existing 197 MHz rf system with 4.5 MV voltage should be able to keep proton beam away from microwave instabilities, which agree with the Keil-Schnell criteria.