SPINOR BASED CALCULATION OF DEPOLARIZING EFFECTS IN CIRCULAR LEPTON ACCELERATORS*

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Abstract

The emission of synchrotron radiation strongly influences the beam dynamics in case of ultra relativistic leptons. When storing or accelerating leptons in circular accelerators, the acting magnetic field shows an oscillating behavior in the rest frame of the leptons. Its properties can be determined by a spectral analysis. This spectrum can be used to simulate depolarizing effects in circular accelerators.

Our contribution will present a calculation of the relevant parts of the mentioned spectrum and a spinor-based determination of the resulting population of the spin-up state including quasi stochastic synchrotron motion. These calculations are based on the lattice of the electron stretcher accelerator (ELSA, Bonn) and confirm the experiences made when observing a non complete reversal of the polarization for high resonance strength.

INTRODUCTION

Currently, many algorithms to calculate spin dynamics in accelerators are based upon orbit and spin tracking. This kind of calculation is in particular suitable for implementing synchrotron radiation in a stochastical way. However, due to this implementation, the computing time increases strongly even if the number of emitted photons is reduced[1]. Thus, the simulation of depolarizing effects for energy ramping lepton accelerators places high demands on the time efficiency of the used algorithm.

An alternative to spin-orbit tracking could be a spinor based calculation. The well known and proved formalism (see [1] for more details) can be used to analyze the influences of disturbing fields during the ramping phase according to Froissart-Stora and will allow to compare the results to those measured at the electron stretcher facility ELSA in future.

In the rest frame of the leptons, the fields generated by the magnetic elements of the accelerator oscillate with various frequencies. Most of these magnetic field oscillations are irrelevant regarding depolarizing effects. If only relevant oscillations are considered, the computing efficiency can be enhanced. By this, the Fourier transformed, then filtered and finally back transformed magnetic fields are used to solve the equation of spinor motion. The resulting enhanced computing efficiency enables a detailed description of the longitudinal phase space via the time dependent Lorentz factor $\gamma(t)$.

The present studies are especially concerned with the boundary conditions of the ELSA stretcher ring. As a first investigation stage, we decide to reduce the transversal phase space macroscopically via taking only the center of charge motion into consideration. In this instance, only integer spin-tune resonances are investigated.

GENERAL APPROACH OF SPINOR CALCULATION FOR LINEAR RAMPING

Exploring a numerical algorithm is associated with a stepwise approach towards the essential phenomena. It is important to show the consistence between numerical results and actually appearing physics and furthermore to abolish any inputs which influence the results in a negligible way. The last step is a major task — since it substantially affects the speed of the procedure — but it also implies special assumptions which might be unsuitable for other cases. Introductory, we describe a total spin-flip at the third integer spin-tune in ELSA without longitudinal synchrotron oscillations.

The spinor motion in accelerators can be described according to an orbit separated Schrödinger equation[2] with

$$\begin{aligned} \partial_t \chi(t) &= \mathrm{i}\frac{e}{2m_0\gamma(t)}\left(1+a\gamma(t)\right)\left(\vec{\sigma}\cdot\vec{B}(t)\right)\chi(t) \\ &= \frac{\mathrm{i}}{2}\,c\left(1+a\gamma\right)\begin{pmatrix} \frac{1}{R} & k\cdot z \\ k\cdot z & -\frac{1}{R}\end{pmatrix}\begin{pmatrix}\chi_+ \\ \chi_-\end{pmatrix} \quad (1) \end{aligned}$$

using the gyromagnetic anomaly of a lepton a as well as the rest energy m_0c^2, unit charge e and the Pauli matrices in $\vec{\sigma}$. This equation holds only for transversally acting magnetic fields which can be described via the bending radius $R(t)$, the quadrupole strength $k(t)$ and the vertical orbit displacement $z(t)$:

$$\begin{pmatrix} B_x(t) \\ B_z(t) \\ 0 \end{pmatrix} \approx \frac{\gamma(t)m_0c}{e}\begin{pmatrix} k(t)\cdot z(t) \\ \frac{1}{R(t)} \\ 0\end{pmatrix}. \quad (2)$$

Here, x indicates the radial and z the vertical direction. Then, the observable degree of vertical polarization is given by its quantum mechanic representation:

$$P_z = {\sigma_z}^{\dagger}\chi(t)\sigma_z = \left|\chi_+\right|^2 - \left|\chi_-\right|^2. \quad (3)$$

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[1] Including all emitted photons in the simulation is not possible. Usually, the stochastic emission of synchrotron light is approximated using so called big photons which are emitted within the middle of a magnetic element and take the integral momentum loss of all emitted photons per element.

[2] This equals to the semiclassical ansatz like it is done for the Thomas-BMT equation. Thus, the self polarizing Sokolov-Ternov effect is neglected as well as Stern-Gerlach spin orbit coupling.

Even though the particle motion is separated from the spin motion in equation (1), the values of the acting fields still depend on the orbit. As a consequence, the periodic motion of the particles and thus also the closed orbit is decisive when describing the phenomena of depolarizing resonances. In general, due to betatron oscillations, the particle motion differs from the closed orbit, which in turn differs from the design orbit due to randomly distributed misalignments of the magnets. Against the real behavior, all particles are assumed to travel along the closed orbit for the present studies. This is a reasonable simplification for integer spin-tune resonances, as all relevantly contributing magnet fields average out, except for those emerging from the motion of the center of charge. Although the transversal single particle motion is delineated in this simplified way, the model is adequate and offers faster computations of the polarization. In Figure 1, the vertical displacements, which are simulated with MAD-X including misalignments, is plotted for one revolution. In order to achieve strong resonance strengths and and a resulting flip of the spins, large displacements are forced by defining strong misalignments. The figure further shows the transversal magnetic field distribution in time and frequency domain.

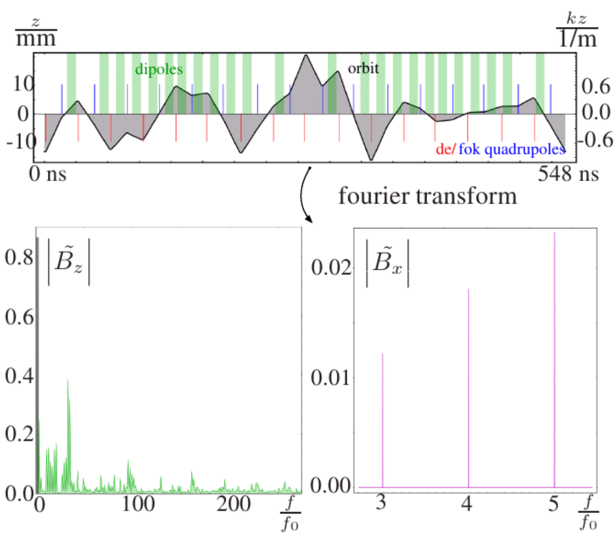

Figure 1: The upper figure shows a disturbed closed orbit, simulated with MAD-X, along a one-turn orbit length of the ELSA stretcher ring. The magnetic fields in the rest frame can be obtained by multiplying the color marked, magnetic field strengths by the vertical displacement. The lower figures show the spectra which result from the Fourier transformation.

In contrast to the transversal single particle phase space, the longitudinal one is of particularly interest while studying integer spin-tune resonances. Initially, a $\gamma(t)$ increasing linearly in time is assumed for each particle. In addition, the total amount of particles shall be Gaussian distributed like it is denoted in Figure 2 using a color code.

Describing the longitudinal phase space in this rudimentary way, the spinor equation can numerically be solved in reasonable computing times even if all information of transversal motion is considered. Whereas in this case the computing efficiency is appropriate, more detailed studies require further enhancing methods. In this context, it became apparent that large parts of the magnetic spectra can be neglected without strongly affecting the results. Then, the computing time is reduced by a multiple of the unfiltered case. For an energy range of 1.2-2.4 GeV, relevantly contributing frequencies are within an interval of approx. 0-6 times the revolution frequency f_0 for both transversal spectra. In this interval, the spin precession frequency $a\gamma f_0$ could be in phase with oscillating transversal fields and thus a depolarization is possible.

Figure 3 shows the solution of equation (1) including the mentioned simplifications for the transversal and longitudinal particle motion.

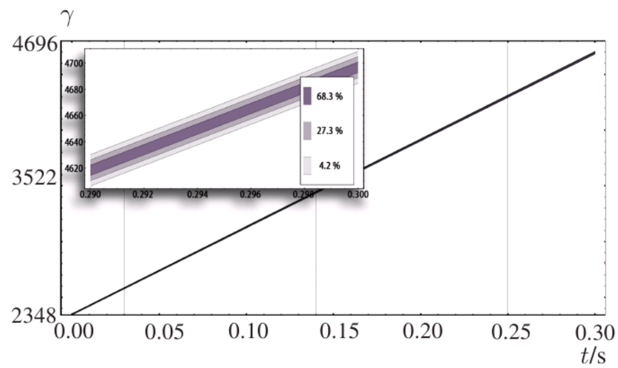

Figure 2: The figure shows the time dependency of the factor $a\gamma(t)$. The $a\gamma$-function is weighted according to the Gaussian shaped energy distribution of the beam. The color range inside the zoomed part of the figure represents the particle density in percent. The graph displays a typical situation at the ELSA stretcher ring. The energy of the electrons is ramped from 1.2 GeV up to 2.4 GeV within 0.3 seconds.

SPINOR CALCULATION WITH SYNCHROTRON MOTION

Actually, the assumption of a strictly linear energy increase as described above is not entirely correct, but the results validate the general approach. Due to the energy deviation, the resonance is crossed at different points in time. For instance, the drop of vertical polarization occurs earlier for particles with positive energy deviations.

Nevertheless, we have to aim for a more realistic description of longitudinal motion including synchrotron oscillation but neglecting single photon emission events.

The currents which excite the magnetic fields of each element increase independently of the particle motion. On closer examination, the Lorentz factor oscillates according to the synchrotron motion of the particles, whereas the currents ramp linearly like the energy of

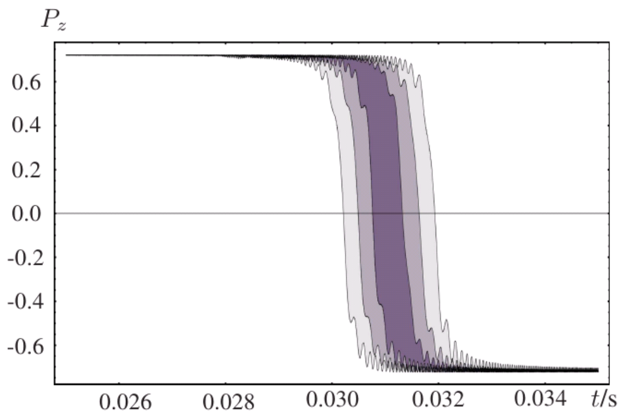

Figure 3: It is shown the result of the numerical solution of equation (1) using explicit Runge-Kutta method. The observed time interval surrounds the third integer resonance $a\gamma = 3$.

the beam does. The particle's Lorentz factor can be defined as: where the linear part of $\gamma^{\star}(t)$ does not differ from the function plotted in Figure 2. In contrast, the oscillating part $\gamma_{\text{osc}}(t)$ is mainly caused by the emission of synchrotron light. The equilibrium of the stochastic excitation and the cavity-induced damping leads to an uncertainty $\Delta\gamma(t)$ (see [2] for detailed describtion) and also the synchrotron oscillation frequencies $\omega_{s,i}(t)$ are distributed unevenly. This is taken into account by a discrete Gaussian weighting via the coefficients a_i. The random character of the longitudinal motion is approximated by adding an uniformly distributed phase ϕ_i. An outline of the longitudinal phase space can be gathered from Figure 4.

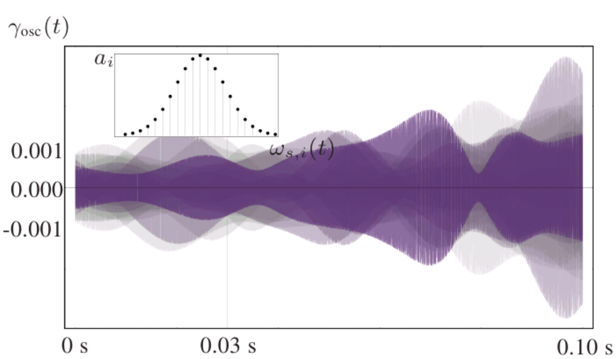

Figure 4: Oscillating part of the Lorentz factor for a number of eight particles. Due to the randomly distributed phases, the envelope of the oscillations varies in a quasi-stochastic manner.

Due to the different behavior of oscillating Lorentz factor of the particles on the one hand and linear current increase on the other hand, the spinor equation (1) has to be modified
The longitudinal oscillations result in synchrotron satellites

Table 1: computing time and deviations assuming different kind of Lorentz factors and neglecting different parts of the vertical magnetic spectrum.

$\gamma_{\text{lin}}(t)$, only zeroth component	$\tau = 1$ AU
$\gamma_{\text{lin}}(t)$, first 6 components	$\tau = 10$ AU
$\gamma_{\text{lin}}(t)$, including all components	$\tau = 548$ AU
$\gamma^{\star}(t)$, first 6 components	$\tau = 301$ AU

in the spectrum. Therefore, further weaker depolarizations occur symmetrically in time both before and after the original polarization drop. The drops, being caused by the synchrotron oscillation, strongly depend on the respective random set of phases ϕ_i and its amplitudes. Indeed, this is unique for a single particle and can also lead to an additional deviation of polarization either positive or negative, but the average of the polarization for many particles converges towards the specific beam polarization (see Figure 5).

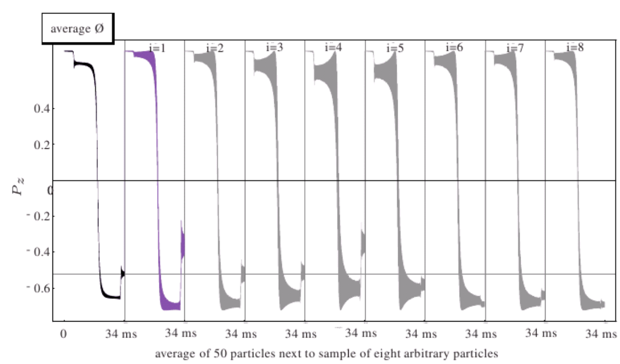

Figure 5: Solution of the numerical callculation of a spin-flip for a sample of eight particles including quasi-stochastic approach of synchrotron oscillations. On the left hand side, the polarization averaged for 50 particles is shown.

By executing the numerical calculations several times and comparing the results, a reproducible converging average polarization could be indicated. Already for averaging the results of 50 particles, the relative deviation of polarization loss is in the lower sub-percent range for all resonance strengths, whether strong or weak. Table (1) subsumes the presented studies. The different computations are classified according to their properties and computing time τ in arbitrary units.

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