Abstract

To conduct experiments using low energy ion beams at the TSR heavy ion storage ring, the beam deceleration process must be well understood. During deceleration of the beam the revolution frequency decreases, resulting in low current, which is difficult to measure with a common DC transformer. However, the number of particles in a bunch can also be determined by measuring the voltage signal in the time domain using a capacitive pick-up. If the ratio of bunch length and RF period does not change during deceleration or acceleration, measuring the pick up signal spectrum is a more sensitive method, where the signal is directly proportional to the number of particles in a bunch. A summary of these different methods to determine the number of particles is presented.

INTRODUCTION

The heavy ion storage ring TSR at the Max-Planck-Institut für Kernphysik in Heidelberg, operates for accelerator, atomic and molecular physics experiments. The storage ring has a circumference of 55.42 m and receives heavy ions from a 12 MV tandem van-de-Graaff and a normal conducting RF linac combination. Light positive ions and negative ions with mass to charge ratio $\frac{A}{q} \leq 9$ are provided by a high-current injector. The TSR is mainly run with ions at injected energy for experiments. However, due to the widely tunable range of the RF resonator an acceleration or deceleration of ions is also possible [1]. The stored ion intensity $I$ is given in terms of the revolution frequency $f_0$, ion charge $q$ and number of ions $N$:

$$ I = q \cdot f_0 \cdot N. $$

As the revolution frequency decreases, the intensity of decelerated ions will be low, which is difficult to measure with the DC current transformer of the TSR. Low intensities, which are below than $1 \mu A$, the detecting limit of current transformers at TSR, require new methods for measuring the ion current.

INTENSITY MEASUREMENT OF A BUNCHED ION BEAM

A current pick-up as shown in fig. 1, consisting of a tube surrounding the stored ion beam, can be used to determine the total intensity of a bunched ion beam by measuring the pick-up voltage $U(t)$. If the bunch length is much larger than the pick-up length $L$, the pick-up voltage can be determined by the following differential equation [2]:

$$ \frac{L}{v} \dot{I}(t) = C \cdot U(t) + \frac{U(t)}{R}, $$

where $v$ describes the ion velocity. The total capacity of the pick-up, the cable connecting the pick-up with the pre-amplifier and the pre-amplifier is denoted by $C$. If the pick-up electrode is loaded with a pre-amplifier with a very high input resistance $R$ the term $U(t)/R$ can be neglected in equation (2) and the pick-up voltage is then directly proportional to the ion beam intensity:

$$ U(t) = \frac{1}{v} \frac{L}{C} I(t). $$

The average stored ion current $\bar{I}$ can be determined by integrating of equation (3) over a RF period:

$$ \bar{I} = C_0 f_0^2 h \frac{C}{L} \int U(t) dt, $$

where $f_0$ is the revolution frequency, $h$ the number of bunches and $C_0$ the circumference of the storage ring. The pick-up signal is measured as a function of beam intensity using $^{12}$C$^{6+}$ ions at $E = 73.3$ MeV. For absolute current measurements the total capacity $C$ has to be known. It can be determined from the measurement of $\int U(t) dt$ as a function of intensity, which is measured with the current transformer, as shown in figure 2. The capacity of $C = 242.7$ pF measured with a LC meter and the value $C = 222.5$ pF obtained from the fit are found to be in a good agreement. Therefore an absolute measurement of the stored ion current is possible by integrating the measured pick-up voltage, taking into account the ion revolution frequency.
Scaling of the the pick-up signal with the ion velocity

According to equation (3) the pick-up signal should scale with the inverse ion velocity. To verify this relationship a carbon ion beam $^{12}\text{C}^{6+}$ was injected, electron cooled and bunched at $E=73.3$ MeV and then accelerated to 240 MeV. During acceleration the integral of the pick-up signal was measured as a function of the revolution frequency $f_0$. With equation (1) and (4) it is possible to calculate the pick-up signal during acceleration:

$$\int U(t)dt = \frac{L}{C} \frac{qN}{C_0 h f_0}.$$  \hspace{1cm} (5)

The efficiency of the acceleration process was determined with the DC transformer. About 98% of the injected ions could be accelerated to the final energy, therefore the number $N$ of ions can be assumed as a constant. The number of ions was determined by the beam current measured at the final energy with the DC current transformer. In figure 3 the integral of the pick-up signal is shown as a function of the revolution frequency. The solid line trough the data is a fit, using equation (5), where the total capacity $C$ was used as an fit parameter. As it is shown in figure 3 the fit describes the data very well, where for the capacity an value of $C = 211.7$ pf was found. Measuring the pick-up signal in the time domain is a tool to exactly measure the efficiency of an acceleration or deceleration process if the ion beam has a minimum intensity of 0.1 $\mu\text{A}$ at velocities of $\beta \approx 0.1$.

Determination of Intensity by measuring the pick-up spectrum

A much more sensitive method to determine the intensity of the stored bunched ion beam is the determination of the pick-up signal spectrum. From equation (3) we can calculate the spectrum of the pick-up voltage $\hat{U}_n$:

$$\hat{U}_n = \frac{1}{v} \frac{L}{C} \hat{I}_n.$$  \hspace{1cm} (6)

The bunch length $w$ of the bunched electron cooled ion beam is space charge limited, having a parabolic longitudinal density profile [1]. The current spectrum $\hat{I}_n$, at the harmonic number $n$ of the RF frequency $f_{rf} (f_{rf} = \frac{n \omega_0}{2 \pi})$, of a beam having a parabolic charge line distribution is determined by the bunch length $w$:

$$\hat{I}_n = \frac{6}{n^3 w^3 \omega_0^3} \left( \sin(n \omega_0) - n \omega_0 \cos(n \omega_0) \right).$$  \hspace{1cm} (7)

The dependency of the bunch length $w$ on the average beam current $\bar{I}$ can be described by [1]:

$$w = C_0 \sqrt{\frac{3(1 + 2 \ln(\frac{2}{3})) \bar{I}}{24 \pi^2 c^4 \epsilon_0 \gamma^2 h^2 \beta^3 U}}.$$  \hspace{1cm} (8)

The bunch length $w$ in formula (8) is determined by the beam intensity $\bar{I}$, the resonator voltage $U$, the number of bunches $h$ in the ring (circumference $C_0$) and the beam velocity $\beta$ in units of the speed of light $c$. The constant $c_0$ is the absolute permittivity and $\gamma$ is the relativistic mass increase (for TSR energies $\gamma \approx 1$). $R$ denotes the radius of the beam tube and $r$ is the average beam radius, defined by the $2\sigma_r$ value of the transverse beam width. From equation (6), (7) and (8) a formula can be derived which can be used to calculate the average beam intensity $\bar{I}$ from the measured voltage spectrum $\hat{U}_n$ of the pick-up signal. Since $w << 1/f_{rf}$ equation (7) can be Taylor expanded, resulting in an approximative formula to calculate the average beam intensity $\bar{I}$ from the measured voltage spectrum amplitude $\hat{U}_1$ at RF frequency $f_{rf}$:

$$\bar{I} = \frac{v C \hat{U}_1}{L^2}.$$  \hspace{1cm} (9)

At the TSR storage ring an experiment was carried out to determine the dependency of the component $\hat{U}_1$ of the pick-up voltage spectrum on the average beam current $\bar{I}$. This measurement was performed with $^{12}\text{C}^{6+}$ ions ($E=50$ MeV). The average beam current was measured with the DC transformer of the TSR storage ring. As shown in figure 4, $\hat{U}_1$ increases linearly at low ion intensities with the stored intensity as predicted by equation (9). The solid red line in figure 4 is a fit through the measurements using equation (6)- (8) where the capacity $C$ was used as a free.

Figure 2: Integral of the measured pick-up voltage as a function of beam intensity.

Figure 3: Measured integral of the pick-up voltage as a function of the revolution frequency during acceleration of a $^{12}\text{C}^{6+}$ beam.
fit parameter. From the fit a value of 224.5 pF could be derived for $C$. The red dashed line is the calculated spectrum $\hat{U}_1$ using equation (9) with 224.5 pF. The capacity from the fit can now be used to determine the beam current for any ion species having different ion velocities. Because the spectrum of the pick-up signal scales with the inverse of the ion velocity (compare equation (9)) this method to determine the absolute intensity of an electron cooled, bunched ion beam is especially sensitive for low velocity ion beams. At beam velocities of $\beta \approx 0.1$ an absolute intensity determination below 10 nA is possible.

Equation (10) is independent of the shape of the bunch if the condition $t_b \ll T_{rf}$ is fulfilled. This is the case if electron cooling is applied. Without electron cooling the shape of the injected ion beam bunch can be approximately by $I(t) \approx 2I_{\text{co}} \cos^2(\pi t/T_{rf})$ resulting in a pick-up signal spectrum at $n = 1$, which is a factor two less compared to the spectrum of a short ion beam, described with equation (10). If electron pre-cooling is used the bunch length during acceleration and deceleration is short and the number of ions during acceleration or deceleration can be approximately determined with equation (10). When the ion beam is accelerated or decelerated the revolution frequency of the ions is changed, shifting the frequency of the spectral line $\hat{U}_1$. To keep the bunch spectrum $\hat{U}_1$ at a constant frequency it is possible to mix the pick-up signal, having the fundamental frequency $f_{rf}$, with a signal which frequency is $f_m = f_{rf} + 2.777$ kHz. This mixing frequency $f_m$ and the RF frequency $f_{rf}$ are generated in a DDS card located in the VME crate of the TSR control system. This mixing process shifts the fundamental frequency of the pick-up signal to 2.777 kHz, easy to measure with a realtime spectrum analyzer. In figure 6 the spectrum $\hat{U}_1$ mixed to 2.777 kHz is shown during acceleration of $^{12}$C$^{6+}$ ions from $E=73.3$ MeV to $E=240$ MeV. The beam was injected at the time $t=0$ s. In the first 1.5 seconds the ion beam was electron cooled recognizable in the change of $\hat{U}_1$. After 1.5 s the cooler was switched off and acceleration starts. At $t = 6.6$ s the final energy was reached and the RF was switched off, indicated at the reduction of $\hat{U}_1$. Because during acceleration there is almost no beam loss (efficiency $\approx 98\%$) and $\hat{U}_1 \sim N$, the signal shown in figure 6 is almost constant. The small deviation can be explained by the bunch length change during acceleration. Therewith we got a very sensitive method to determine approximately the stored ion number during acceleration or deceleration.

Figure 4: Measured pick-up spectrum $\hat{U}_1$ as a function of the current of a carbon beam having a velocity of $\beta = 0.09$.

Figure 5: Mixing of the pick-up signal to 2.777 kHz.

**Determination of the approximate ion number during acceleration and deceleration**

Sometimes deceleration and acceleration has to be carried out with very weak ion beams ($I \ll 1\mu A$). For these weak beams the DC transformer can not be used to determine the number of stored ions, therefore we are looking for other possibilities to measure the number of ions. A very efficient method is the measurement of the pick-up signal spectrum. If the bunch length $t_b$ is very short compared to the RF-period $T_{rf}$, we get from equation (9) the following relationship between pick-up spectrum $\hat{U}_1$ and particle number $N$:

$$\hat{U}_1 = \frac{2q L}{C_0 C} N$$

Equation (10) is independent of the shape of the bunch if the condition $t_b \ll T_{rf}$ is fulfilled. This is the case if electron cooling is applied. Without electron cooling the shape of the injected ion beam bunch can be approximately by $I(t) \approx 2I_{\text{co}} \cos^2(\pi t/T_{rf})$ resulting in a pick-up signal spectrum at $n = 1$, which is a factor two less compared to the spectrum of a short ion beam, described with equation (10). If electron pre-cooling is used the bunch length during acceleration and deceleration is short and the number of ions during acceleration or deceleration can be approximately determined with equation (10). When the ion beam is accelerated or decelerated the revolution frequency of the ions is changed, shifting the frequency of the spectral line $\hat{U}_1$. To keep the bunch spectrum $\hat{U}_1$ at a constant frequency it is possible to mix the pick-up signal, having the fundamental frequency $f_{rf}$, with a signal which frequency is $f_m = f_{rf} + 2.777$ kHz. This mixing frequency $f_m$ and the RF frequency $f_{rf}$ are generated in a DDS card located in the VME crate of the TSR control system. This mixing process shifts the fundamental frequency of the pick-up signal to 2.777 kHz, easy to measure with a realtime spectrum analyzer. In figure 6 the spectrum $\hat{U}_1$ mixed to 2.777 kHz is shown during acceleration of $^{12}$C$^{6+}$ ions from $E=73.3$ MeV to $E=240$ MeV. The beam was injected at the time $t=0$ s. In the first 1.5 seconds the ion beam was electron cooled recognizable in the change of $\hat{U}_1$. After 1.5 s the cooler was switched off and acceleration starts. At $t = 6.6$ s the final energy was reached and the RF was switched off, indicated at the reduction of $\hat{U}_1$. Because during acceleration there is almost no beam loss (efficiency $\approx 98\%$) and $\hat{U}_1 \sim N$, the signal shown in figure 6 is almost constant. The small deviation can be explained by the bunch length change during acceleration. Therewith we got a very sensitive method to determine approximately the stored ion number during acceleration or deceleration.

Figure 6: Measured first harmonic of the pick-up signal, mixed to 2.777 kHz, during acceleration of a $^{12}$C$^{6+}$ beam.

**REFERENCES**
