INVESTIGATION OF BEAM LOADING EFFECTS FOR THE NEUTRINO FACTORY MUON ACCELERATOR

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Abstract
The International design study (IDS) study showed that a Neutrino Factory [1] seems to be the most promising candidate for the next phase of high precision neutrino oscillation experiments. One part of the increased precision is due to the fact that in a Neutrino Factory the decay of muons produces a neutrino beam with narrow energy distribution and divergence.

The effect of beam loading on the energy distribution of the muon beam in the Neutrino Factory decay rings has been investigated numerically. The simulations have been performed using the baseline accelerator design including cavities for different number of bunch trains and bunch train timing.

A detailed analysis of the beam energy distribution expected is given together with a discussion of the energy spread produced by the gutter acceleration in the FFAG and the implications for the neutrino oscillation experiments will be presented.

NEUTRINO FACTORY BASELINE
The influence of beam loading on the energy distribution delivered by the muon accelerator of an NF has been discussed at the IDS NF plenary meetings in CERN and Mumbai as well as in Chicago. To quantify the changes of the energy distribution within the bunch trains due to beam loading, assuming different scenarios under consideration, a numerical model of the accelerator chain shown in figure 1 has been produced and simulations for different scenarios performed.

DEFINITION OF RF PARAMETERS
The linac consists of 3 families of cryo modules each consisting of one solenoid and one, two and four SC cavities at 201.25 MHz. The total number of cavities is 66 and the average energy gain is ~10 MeV per cavity. The RLA 1 consists of 24 Modules with 2 cavities each (= 48 cavities) producing the required total acceleration in the simplified model within 4.5 passes with an average energy gain of ~12.5 MeV per cavity. RLA 2 consists of 80 Modules with 2 cavities each (= 160 cavities total) producing the required total acceleration in 4.5 passes with an average energy gain of ~12.5 MeV per cavity. For the FFAG three different designs are under consideration. An energy gain of ~12.5 MeV per cavity is assumed for the FFAG. The main properties of the three designs are summarized in table 1. While the FCDC design minimizes the beam loading (BL) effect for the simulations the FDFCC design has been used as this design is preferable in terms of available space in the drift sections.

Table 1: Summary of FFAG Design Parameters for The Investigation of Beam Loading Effects.

<table>
<thead>
<tr>
<th></th>
<th>FDFC</th>
<th>FCDC</th>
<th>FDFCC</th>
</tr>
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<tbody>
<tr>
<td>Number of cells</td>
<td>80</td>
<td>68</td>
<td>64</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>519</td>
<td>521</td>
<td>546</td>
</tr>
<tr>
<td>Energy gain per turn (MV)</td>
<td>763</td>
<td>1475</td>
<td>1119</td>
</tr>
<tr>
<td>Number of turns</td>
<td>17.3</td>
<td>8</td>
<td>11.8</td>
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NF particle numbers in the muon accelerator
To achieve the goal defined by particle physics of $10^{21}$ useful muon decays per year of 10$^7$ sec under assumption of a decay ring efficiency of 37% a total number of $5.4\times10^{12}$ muons must be delivered by the accelerator at 50 Hz. The NF baseline assumes bunch trains of 88 bunches delivered by the muon front end with 1 or 3 bunch trains per 50 Hz delivered. Derived from these figures the number of muons per bunch has been assumed to be $6.2\times10^9$ particles for one bunch train and $2.07\times10^{10}$ for the 3 bunch train scenario. While for the one bunch train (at 50 Hz) scenario the energy spread due to beam loading is defined by the energy gain of the particles only, in the multi bunch train scenarios the effect on the following bunch trains can be reduced by the amount of available RF power and the inter bunch train timing.

RF cavities and stored energy
As the beam loading effect is strongly dependent on the energy stored in the cavity a detailed analyses of the properties of different single cell cavity designs has been performed [4] and the results are summarized in table 2.
Table 2: Summary of RF Cavity Properties for The Investigation of Beam Loading.

<table>
<thead>
<tr>
<th></th>
<th>$\beta=0.9$</th>
<th>$\beta=1$</th>
<th>Study II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy gain (MV)</td>
<td>9.00</td>
<td>8.61</td>
<td>8.84</td>
</tr>
<tr>
<td>L (m)</td>
<td>0.67</td>
<td>0.7448</td>
<td>0.8282</td>
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<tr>
<td>Stored energy (J)</td>
<td>772</td>
<td>712</td>
<td>747</td>
</tr>
<tr>
<td>el. peak field (MV/m)</td>
<td>27.2</td>
<td>26.2</td>
<td>26.4</td>
</tr>
</tbody>
</table>

**SIMULATION RESULTS**

Every cavity in the simulation was filled to the required field level before the first bunch was started. In the case of several bunch trains after each bunch train the cavities are refilled by the RF according to RF power and the time spacing between bunches. The following cases have been investigated: Case 0: No beam loading (= reference), case 1: 1 bunch train full beam loading, case 2: 3 bunch trains full BL, case 3: 5 bunch trains full BL, case 4: 3 bunch trains 250 kW coupler, 10 $\mu$s bunch train spacing, case 5: 3 bunch trains 250 kW, 50 $\mu$s, case 6: 3 bunch trains 250 kW, 100 $\mu$s, case 7: 3 bunch trains 500 kW, 100 $\mu$s, case 8: 3 bunch trains 1 MW coupler 100 $\mu$s bunch spacing.

The result of the simulations are summarize in figure 2 where the evolution of the energy of all bunches in the bunch train(s) from the muon front end to the decay rings for different scenarios is shown.

In figure 3 the (average) particle energy of a bunch within the bunch train is shown for the initial particle distribution delivered by the front end (244MeV) and after the first acceleration stage behind the linac (900MeV). Obviously the additional energy spread due to beam loading in the linac is minor compared to the initial energy spread at injection. A more detailed picture of the situation at the end of the linac is shown in figure 4.

While the bunch train remains mono energetic without any beam loading, the one bunch train scenario shows an energy spread along the bunch train of ~5MeV, and the same total spread (over more bunches in total) is seen for the 3 and 5 bunch train case without RF. In all other cases beam loading is significantly less and negligible compared with the initial energy spread.

Figure 2: Energy development of bunch trains in the muon accelerator (section 1 = linac, section 2-6 RLA1, section 7-11 RLA2, section 12-23 FFAG), for different number of bunch trains considering beam loading, RF coupling and different time spacing of the bunch trains.

Figure 3: Energy of bunches along the bunch train at the beginning and the end of the muon linac. The initial energy spread of the beam is to be $\pm$ 10%.

Figure 4: Energy distribution along the bunch train at the end of the muon linac. The scale of the graph is equivalent to the initial energy spread of $\pm$24MeV.

Figure 5: Detailed view of the energy distribution along the bunch train at the end of the first RLA.
In figure 5 the detailed picture of the energy distribution at the end of RLA1 is shown. While the bunch train remains mono energetic at nominal beam energy in the case without any beam loading, the one bunch train scenario shows a significant loss of energy of ~30 MeV, for the first bunch and a similar spread over the bunches. For the 3 bunch train scenario the situation differs from before. While the first bunch train is shifted in energy by ~10 MeV (1/3 of 1 BT case) and the energy spread in the fist bunch is also 1/3 the difference for the case 1. At the end of the RLA1 the different scenarios concerning the RF power and bunch train timing become obvious. Without any energy in the cavities recovered between the bunch trains the successive bunch trains suffer each time an additional shift in energy while the energy spread within the bunch train is limited to one third of the one train case. This means that the central bunch in both cases has the same energy and the sum of the inter bunch train energy spread is the same, but due to the additional energy shift the three individual bunch trains spread over a larger regions of energy. In the cases where a part or all energy is recovered between the bunch trains no major additional energy shift is noticeable.

In figure 6 the detailed picture of the energy distribution at the end of RLA2 is shown. The situation is pretty much the same compared to RLA1. The main differences are visible between the different RF cases. While the cases 9 and 8 still fully recover all energy in the cavities between the bunch trains and the energy spread due to beam loading is ~1/3 of the one bunch train case. Case 7 is equivalent with the one bunch train scenario and all others show larger energy spread over all bunches. In figure 7 the detailed picture of the energy distribution at the end of the FFAG is shown. The situation is pretty much the same compared to RLA2 but the effects further increased. The major difference is visible in the case 8 and 9. While in the while in case 8 the energy gap increases further and the total energy spread now exceeds the values for case 1, the overlap of the bunch energies for different trains in case 9 is reduced to zero and the total energy spread only little smaller than for case 1. Some of the main values concerning the energy shift and spread are summarized in table 3.

**CONCLUSIONS**

While for a non chromaticity corrected lattice the energy spread induced by the large amplitude particles is ±2.5% (1250 MeV) and exceeding any energy spread due to beam loading this is not the case for a (partly) chromaticity corrected lattice [5]. To keep the energy spread due to beam loading comparable small to the energy spread by the acceleration of large amplitude particles 1 MW couplers and a bunch train spacing of 100 μs has been adopted to be the new base line values.

**REFERENCES**