Polarization in SuperB*

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Abstract

SuperB, the 2nd-generation B-Factory with a luminosity of $10^{36}$ cm$^{-2}$/s proposed for LNF, is being designed from the start to be capable of providing a spin-polarized electron beam in the low-energy ring (LER) with longitudinal polarization at the interaction point.[1] Due to the high luminosity at moderate beam current the beam lifetime is short (a few minutes), and a polarized injector will be used. Spin rotators have been designed and the equilibrium polarization evaluated. It will be shown that an average polarization of about 70% can be expected.

INTRODUCTION

Relativistic electrons circulating in the guide field of a storage ring emit synchrotron radiation and a tiny fraction of the photons can cause spin flip from “up” along an initial direction to “down”, and vice versa. However, the up-to-down and down-to-up rates differ, with the result that the beam can become spin polarized. In a perfectly aligned flat ring with no solenoids the polarization is anti-parallel to the vertical guide field, reaching a maximum polarization, $P_{st}$, of $\frac{8}{5\sqrt{2}} = 92.4\%$. This, the Sokolov-Ternov (S-T) polarizing process[2], is very slow on the time scale of other dynamical phenomena occurring in storage rings, and the inverse time constant for the exponential build up in a uniform dipole field is[2]:

$$\tau_{st}^{-1} = \frac{5\sqrt{3} \gamma^5}{8 m_e}$$

where $r_e$ is the classical electron radius, $\gamma$, the Lorentz factor, $\rho$, the bending radius in the guide field, and the other symbols have their usual meanings.

In a simplified picture the majority of the photons in the synchrotron radiation do not cause spin flip but tend instead to randomize the orbital motion in the magnetic fields due to the presence of dispersion. Then the spin-orbit coupling embodied in the Thomas-BMT equation can cause spin diffusion, i.e. depolarization. The equilibrium polarization is then less than 92.4% and will depend on the relative strengths of the polarization and depolarization processes.

Analytical estimates of the attainable equilibrium polarization are best based on the Derbenev-Kondratenko (D-K) formalism.[3, 4] The equilibrium polarization is given by

$$P_{dk} = \frac{8}{5\sqrt{3}} \frac{\int ds \left< \hat{r} \cdot (\hat{n} - \frac{\hat{m} \hat{b}}{\|\hat{m}\|}) \right>}{\int ds \left( 1 - \frac{2}{9} (\hat{n} \cdot \hat{s})^2 + \frac{11}{18} \left(\frac{\hat{m} \cdot \hat{b}}{\|\hat{m}\|}\right)^2 \right)}$$

where $< >$ denotes the average over phase space at azimuth $s$, $\hat{s}$ is the direction of motion, and $\hat{b}$, the magnetic field direction. Furthermore, $\hat{n}$ is a unit 3-vector satisfying the Thomas-BMT equation along particle trajectories and is 1-turn periodic. The ensemble of $\hat{n}$-vectors for a beam is called the invariant spin field. On the closed orbit, i.e. at the origin in phase space, $\hat{n}$ becomes $\hat{n}_0$.[5] In conventional situations in electron rings, $\langle \hat{n} \rangle$, is very nearly aligned with $\hat{n}_0(\rho)$. In the presence of radiative depolarization, the rate given above must be replaced by

$$\tau_{dk}^{-1} = \frac{5\sqrt{3} \gamma^5}{8 m_e} \frac{1}{C} \int ds \left( 1 - \frac{2}{9} (\hat{n} \cdot \hat{s})^2 + \frac{11}{18} \left(\frac{\hat{m} \cdot \hat{b}}{\|\hat{m}\|}\right)^2 \right)$$

the quantity $d^2 = (\frac{\hat{m} \cdot \hat{b}}{\|\hat{m}\|})^2$ is a key parameter in evaluating the expected polarization. Large values of $d^2$ cause low equilibrium polarization $P_{ens,\tau_{dk}}(\rho)$ and small time constants $\tau_{dk}$, thus reducing the polarization attainable. $d^2$ can become very large at the spin-orbit resonances where the spin tune—which is $\gamma G$ for a flat ring—equals an integer or a harmonic of the machine tunes.[5] Here, $G = (g-2)/2$ is the gyromagnetic anomaly, $G \approx 0.00116$ for electrons.

SuperB

Quantitative evaluation of $\tau_{st}$ for SuperB gives about 5...7 hours for either ring, which is too long to be useful as a polarization mechanism. Therefore SuperB will achieve polarized beams by injecting polarized electrons into the LER. We chose the LER rather than the HER because the spin rotators (see below) employ solenoids. These rotators are more compact than dipole-based spin rotators, but the solenoids need to be scaled in strength with energy thus making LER spin rotators more compact than spin rotators in the HER would be.

In SuperB at high luminosity the beam lifetime will be only 3...5 minutes and continuous injection (“trickle-charge”) operation is a key component of the proposal. By injecting at a high rate with a polarized beam one can over-ride the depolarization in the ring as long as the depolarization time constant is not too small. The equilibrium polarization under continuous injection is given by

$$P = P_t \frac{\tau_{dk}}{\tau_{dk} + \tau_b} + P_{dk} \frac{\tau_b}{\tau_{dk} + \tau_b}$$

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01 Circular Colliders
A02 Lepton Colliders
Table 1: Parameters of Spin Rotator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Energy</td>
<td>4.18 GeV</td>
<td></td>
</tr>
<tr>
<td>Spin rotation of solenoids</td>
<td>90°</td>
<td>one side</td>
</tr>
<tr>
<td>Solenoid field integral</td>
<td>4*10.94 Tm</td>
<td>4 solenoids</td>
</tr>
<tr>
<td>Solenoid field</td>
<td>2.39 T</td>
<td></td>
</tr>
<tr>
<td>Tot. length of solenoid section</td>
<td>23.07 m</td>
<td>incl. de-coupling</td>
</tr>
<tr>
<td>Spin rotation of dipoles</td>
<td>270°</td>
<td>one side</td>
</tr>
<tr>
<td>Bending of dipoles</td>
<td>28.4°</td>
<td>one side</td>
</tr>
</tbody>
</table>

where $\tau_b$ is the beam lifetime in the ring and $P_i$, the polarization of the injected beam. As long as $\tau_b < \tau_{dk}$, the first term dominates (for high $P_i$).

**Spin Rotators**

In the arcs of the ring, the vector $\hat{n}_0$, which gives the direction of the polarization, must be close to vertical to minimize depolarization. In order to obtain longitudinal polarization at the IP, a net rotation of $\hat{n}_0$ by 90° about the radial axis is required. A rotation of 90° in a solenoid followed by a spin rotation of 90° in the horizontal plane provides the required net rotation about the radial axis without any vertical bending—thus avoiding vertical emittance growth—and is adopted for SuperB. After the IP, $\hat{n}_0$, and with it the polarization, has to be restored to vertical by a second spin rotator. Two geometries are possible: an antisymmetric geometry where the dipoles and solenoids after the IP have polarities opposite to those before the IP and a symmetric geometry, where the polarities are all the same. The two solutions have significantly different properties[6]:

- With the antisymmetric geometry and perfect alignment, $\hat{n}_0$ is vertical in the arcs at all energies; the rotators are to lowest order *spin matched* in energy.
- With the symmetric geometry, $\hat{n}_0$ is vertical in the arcs and longitudinal at the IP at just one energy and the whole interaction region will normally *not* be spin matched in energy.

For SuperB at high luminosity, the LER beam lifetime is about 3…5 min. Under these conditions it turns out that a symmetric spin-rotator scheme is feasible and can achieve 70% polarization or better. For the solenoids to avoid coupling the betatron planes, we employ the decoupling scheme worked out by Zholents and Litvinenko[7], which inserts quadrupoles in between two half-solenoids to invert one of the planes, thus decoupling each full spin-rotator solenoid.

**LER Spin Rotator Layout**

Fig. 1 shows the IR of the LER with spin rotators. The rotator parameters are given in Table 1. Note that the dipole section rotates the spin by 270° instead of 90°; this was done in order to integrate the rotators with the local chromaticity correction needed in the IR. With 90° dipole angle, the total bending would have been too small and the dispersion insufficient for effective chromaticity correction.

**Spin Dynamics**

In order to evaluate the rate of depolarization we used the code SLICKTRACK. This code is an extension of the code SLICK which can perform analytic first-order thick-lens evaluations using the SLIM[8] formalism. The extension comprises a Monte-Carlo spin-orbit tracking algorithm for simulating full 3-d spin-orbit motion in the presence of synchrotron radiation.

The following results are based on the MAD lattice model of the LER to which accelerating cavities have been added. A limited set of misalignments (in the arcs only) was implemented. Orbit correction was done in SLICKTRACK using a reduced set of correctors. SLICKTRACK calculates beam emittances and the values obtained are close to the design values in the horizontal plane, while larger than the design in the vertical plane (due to limited attempts at orbit correction). The energy spread and synchrotron tune are close to the design values. The differences w.r.t. the MAD parameters are explained by the different treatment of RBENDs in MAD vs SLICKTRACK (MAD takes the length as hardware length and then calculates the orbit length, whereas SLICKTRACK takes the length as orbit length) as well as the different misalignment and orbit-correction setups.

Fig. 2 shows the (de-)polarization time $\tau_{dk}$ vs ring energy (which is $0.441 \cdot \gamma G$ [GeV]). The behavior seen mainly reflects the variation in $d^2$ in the arcs as the angle of tilt of $\hat{n}_0$ from the vertical in the arcs varies with the ring energy as a result of using the symmetric rotator. Moreover since the interaction regions are not spin transparent, $d^2$ is not small even when $\hat{n}_0$ is vertical in the arcs. Consequently $\tau_{dk}$ is always much smaller than the Sokolov-Ternov time. The maximum equilibrium polarization (without continu-

![Figure 2: Depolarization time vs ring energy in the LER.](image-url)
Figure 1: Interaction Region layout with spin rotators. The arrows indicate the orientation of the $\hat{n}_0$ axis.

Figure 3: Average polarization in the LER under continuous injection and at 3.5 min. beam lifetime.

Figure 4: Tilt of the $\hat{n}_0$ axis against the beam direction at the IP. The longitudinal component is plotted.

REFERENCES


SUMMARY

The results obtained so far give confidence that a polarization in excess of 70% at high luminosity can be achieved in the SuperB LER with the injection of polarized beams. Further studies will focus on the assessment of the need to improve the spin matching[5] of the interaction region including the rotator solenoid sections, the effect of imperfections including residual detector solenoid fields, and spin-tracking to include the higher order and beam-beam effects. The preservation of the polarization during the injection process will also be studied.