S-POD EXPERIMENTS
OF SPACE-CHARGE-DOMINATED BEAM RESONANCES

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Abstract
S-POD (Simulator for Particle Orbit Dynamics) is a tabletop, non-neutral plasma trap system developed at Hiroshima University for fundamental beam physics studies. The main components of S-POD include a compact radio-frequency quadrupole (RFQ) trap [1], various AC and DC power supplies, a vacuum system, a laser cooler, several diagnostics, and a comprehensive computer control system. A large number of ions, produced through the electron bombardment process, are captured and confined in the RFQ trap to emulate collective phenomena in space-charge-dominated beams traveling in periodic linear focusing lattices. This unique experiment is based on the isomorphism between a one-component plasma in the laboratory frame and a charged-particle beam in the center-of-mass frame [2]. We here employ S-POD to explore the coherent betatron resonance instability which is an important issue in modern high-power accelerators. Ion loss behaviors and transverse plasma profiles are measured under various conditions to identify the parameter-dependence of resonance stop bands.

S-POD

The \( \mu \)-space distribution function \( f \) of the charged particles confined in an RFQ trap obeys the Vlasov equation \( df \equiv \frac{\partial f}{\partial t} + [f, H] = 0 \), where \([,] \) denotes the Poisson’s bracket and \( H \) is the Hamiltonian that governs the particle motion. Assuming an axially uniform plasma column for simplicity, \( H \) takes the form

\[
H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(t)(x^2 - y^2) + \frac{q}{mc^2} \phi, \tag{1}
\]

where \( m \) and \( q \) are the rest mass and charge state of the particle, \( c \) is the speed of light, the space-charge scalar potential \( \phi \) satisfies the Poisson’s equation, and \( K(t) \) is proportional to the RF voltages applied to the trap electrodes. The closed set of the equations described above is physically equivalent to that governing the collective behavior of a coasting beam in a linear transport channel [2]. The compact one-component plasma trap system can thus be a powerful experimental tool that enables us to explore diverse beam dynamic phenomena in particle accelerators.

The RFQ trap employed for the present resonance study is illustrated in Fig. 1. The quadrupole rods are axially divided into three pieces, so that we can form a couple of potential wells for longitudinal plasma confinement. The diameter of the ion trapping region surrounded by the quadrupole electrodes is 1 cm and the total axial length of the trap is less than 20 cm. In the resonance experiments discussed below, we confine ion plasmas in the Ion Source (IS) region by applying DC biases to an end cap (Cap A) and the short central quadrupole (Gate). A microchannel plate (MCP) with a phosphor screen and a Faraday cup are placed at both sides of the trap to measure the transverse profile and total charge of the plasma. Detailed information of the whole S-POD system has been given in ref. [3]. The non-neutral plasma physics group in Princeton has constructed a similar ion trap intended for beam dynamics studies [4].

COHERENT RESONANCE

Resonance instability is driven by the periodicity of the external focusing field. The beam motion as a whole is the superposition of various collective oscillation modes whose tunes depend on the charge density and focusing strength. When the tune of a collective mode satisfies a certain relation with one of the driving-field harmonics, that mode is resonantly excited leading to a beam loss. The simplest 1D criterion for resonance instability (the coherent resonance condition [5]) is given by

\[
m(v_0 - C_w \Delta v) = N_{\text{sp}} \cdot \frac{n}{2}, \tag{2}
\]

where \( v_0 \) is the bare betatron tune, \( \Delta v \) is the tune shift induced by the space-charge potential, the collective-mode number \( m \) and the driving-harmonic number \( n \) are both positive integers, \( N_{\text{sp}} \) is the superperiodicity of the focusing lattice, and \( C_w \) is a mode-dependent constant smaller than unity. Note the factor 1/2 on the right hand side which is missing in the original coherent resonance condition derived by Sacherer [6]. The condition in Eq. (2) indicates that a resonance of the \( m \)th order occurs when the corresponding collective tune is close to a half integer (not an integer). For example, a linear (second order, \( m = 2 \)) resonance can be excited at a bare tune not only near a half integer but also near a quarter integer.

Figure 1: Layout of the ion trap.
**EXPERIMENTAL RESULTS**

The RF power source for S-POD can generate arbitrary periodic waveforms that imitate a wide variety of linear focusing lattices. The fundamental properties of coherent resonances can, however, be studied systematically without the use of complex pulse excitations of the electrodes because only one of many harmonics composing the periodic lattice function $K(t)$ is responsible for a particular resonance. In fact, Eq. (2) is the condition of $m$th-order resonance driven by the $n$th harmonic; no other harmonics contributes to this resonance. In the present experiments, therefore, we simply employ a few sinusoidal RF waveforms that have different frequencies [7]. $\text{Ar}^{+}$ plasmas are first produced in the IS by ionizing neutral Ar gas with an electron gun, confined there typically for 1 - 10 ms, and finally extracted toward either the Faraday cup or MCP for ion detection. The primary focusing RF field has the frequency of 996 kHz. Then, the RF amplitude of 93 V is enough to survey the whole betatron tune range.

**Resonance Stop bands**

As an example, we here assume a storage ring consisting of 12 FODO cells. Since a single sinusoidal period of the focusing RF field corresponds to a single FODO cell, the revolution frequency is $996/12 = 83$ kHz.

Without other sinusoidal potentials added to the quadrupole electrodes, the focusing field over one turn around the ring is said to have perfect 12-fold symmetry ($N_{sp} = 12$). The resonance condition is then

$$v_0 - C_m \Delta v = 6n/m.$$  

(3)

The broken line in Fig. 2 shows the experimental data corresponding to this highly symmetric case. Two major stop bands are observed at $v_0 = 2.2$ and 3.2. According to Eq. (3), the former is generated mainly by a nonlinear ($m = 3$) resonance and the latter mainly by a linear ($m = 2$) resonance. Nonlinear resonances are probably enhanced in the RFQ trap because of the inevitable error field originating from the non-ideal geometry and misalignments of the electrodes.

We now superimpose a perturbative RF potential of $83 \times 3 = 249$ kHz to reduce the strict periodicity of the driving force. This means that the ring is composed of three lattice superperiods ($N_{sp} = 3$), each of which contains four FODO cells. The amplitude of the $N_{sp} = 3$ perturbation is fixed at 0.5% of the primary focusing-field amplitude. The resonance condition turns to

$$v_0 - C_m \Delta v = 3n/2m.$$  

(4)

The experimental results of this case are plotted with the solid line in Fig. 2. Several additional stop bands have appeared around $v_0 = 0.8$ and 1.7 while the two instability regions observed in the $N_{sp} = 12$ case are still there unchanged. The stronger instabilities that have caused sharper ion losses at $v_0 = 0.75$ and 1.6 are due to the second-order ($m = 2$) resonance. Each of them is accompanied by a weaker but wider stop band expected to be mainly fourth order ($m = 4$). In theory, $C_2 < C_4$ [5,6], which should be why the split of the linear and nonlinear stop bands has occurred. The observed $m = 3$ and $m = 4$ stop bands (even the $m = 2$ stop band at $v_0 = 3.2$ as well) may involve weak higher-order resonances.

The root-mean-squared tune depression is about 0.8 when the number of confined ions in the IS is near $10^7$. The initial plasma density can be controlled to some degree by changing the neutral Ar gas pressure, ionization time, and electron-beam current. Figure 3 demonstrates the effect of space-charge-induced detuning. As the initial intensity of the plasma increases, all stop bands shift to the higher tune side (rightward). The amounts of detuning appear to be slightly different for stop bands of different orders, which is again attributed to the $m$-dependence of $C_m$. The two adjacent stop bands ($m = 2$ and 4) slightly above $v_0 = 1.5$ approach each other at lower intensity and eventually overlap.

**Profile Measurements**

By switching off the DC bias potential on the Cap A (see Fig. 1), we can launch the plasma toward the MCP detector, thus enabling transverse profile measurements with the phosphor screen. A high-resolution CCD camera is put beside the screen to measure fluorescence signals.
A couple of sample data are displayed in Fig. 4 for reference. The left and right pictures were taken, respectively, after 0.2-msec and 0.6-msec storage of the plasma in the IS. The operating conditions of the trap are identical in both cases. The bare tune is set at $\nu_0 = 1.6$ with the $N_{sp} = 3$ perturbation RF field on, which means that the plasma is inside a linear stop band according to Fig. 2. We recognize the distortion of the plasma profile as well as the fluorescent intensity reduction in the right picture.

Figure 5 shows the time evolution of radial charge-density profiles measured every 0.2 msec under different experimental conditions. The total ion number is close to $10^7$ in all cases initially ($t = 0$). The left two data are obtained at $\nu_0 = 1.6$ while the right two at $\nu_0 = 1.8$. In the upper two cases, the driving force holds the strict 12-fold symmetry ($N_{sp} = 12$). Since the plasma is out of resonance at both tune values as confirmed in Fig. 2, the plasma is quite stable over 1.0 msec. The tunes are unchanged in the lower two cases, but now the $N_{sp} = 3$ perturbation is turned on. Both operating points are then deep inside the $m = 2$ and $m = 4$ stop bands (see the solid line in Fig. 2), which considerably weakens the CCD signals due to major ion losses.

**SUMMARY**

We have developed the novel experimental tool “S-POD” that enables us to explore a wide range of beam physics issues [7]. S-POD is just tabletop size and far cheaper than any conventional system relying on large-scale accelerators or beam transport channels. The extremely high tunability of fundamental parameters makes it possible to conduct systematic investigations of various beam instability mechanisms depending on tunes, lattice designs, beam densities, bunch configurations, mismatches, etc. Since the plasma is at rest in the laboratory frame, it is much easier to obtain detailed and accurate experimental data. Unlike accelerator-based experiments, we do not have to worry about radioactivation due to particle losses. S-POD experiments are totally computer-controlled and, therefore, automatically executed without human intervention to retune parameters. Although the experimental result shown in Fig. 3 is based on nearly 1000 independent measurements, it took us only a few hours to accumulate the data.

Among diverse applications of S-POD, we here chose coherent betatron resonance that is of particular importance in high-power or low-emittance machines. By superimposing RF quadrupole fields of different frequencies, we have studied the lattice-induced resonance instability. The excitation of linear and nonlinear resonance stop bands depending on lattice periodicity is experimentally demonstrated. It is shown that a coherent resonance takes place when the tune of the corresponding collective mode is close to a half integer.

**ACKNOWLEDGEMENTS**

This work was supported in part by a Grant-in-Aid for Scientific Research, Japan Society for the Promotion of Science, by the High Energy Accelerator Research Organization, and by Lawrence Livermore National Laboratory under US Department of Energy contract No. DE-AC52-07NA27344.

**REFERENCES**