NEW APPROACHES TO MUON ACCELERATION WITH ZERO-CHROMATIC FFAGS

T. Planche*, J-B. Lagrange, E. Yamakawa,
Graduate School of Engineering, Kyoto University, Japan
Y. Ishi, Y. Kuriyama, Y. Mori, K. Okabe, T. Uesugi,
Kyoto University Research Reactor Institute, Kumatori, Japan

Abstract

The acceleration of intense muon beams up to 25 GeV is the challenge of the international design work for a future neutrino factory. The present baseline scenario for muon acceleration is based on linacs, recirculating linear accelerators (RLAs) and non-scaling fixed field alternating gradient (FFAG) rings. However RLA are one of the most cost driving part. Two new approaches to use zero-chromatic FFAG instead of RLA have been proposed. Detailed lattices parameters and 6D tracking results are presented.

INTRODUCTION

The production of high-energy and high-intensity muon beams for future neutrino factories requires to accelerate, in a very short time, large 6D emittance beams [1, 2]. Among the different possible acceleration schemes, the scaling type of fixed field alternating gradient (FFAG) ring has remarkable features. Since betatron tunes stay constant during acceleration, particle can be kept far from harmful resonances. Moreover, since the chromaticity is null, the first order dependence of the time of flight on the transverse amplitude disappears [3]. Scaling FFAGs are then weakly affected by the longitudinal amplitude growth for large transverse amplitude. This makes all the more large the 6D acceptance achievable with this type of ring.

The use of scaling FFAGs for muon acceleration has already been proposed [1]. But designs presented in Ref. [1] assumed variable frequency acceleration at frequencies of the order of 5 MHz, which is incompatible with the present baseline design [4]. However two ways to use constant rf frequency acceleration with scaling FFAGs have recently been proposed [5, 6]. In this paper we thus reexamine the possibilities offered by scaling FFAGs, considering the harmonic number jump (HNJ) acceleration on one side, and the stationary bucket (SB) acceleration on the other side.

HARMONIC NUMBER JUMP ACCELERATION

The principle of the HNJ acceleration is to keep the synchronization between the revolution period \( T_{\text{rev}} \) of a reference particle and the rf period \( T_{\text{rf}} \):

\[
T_{\text{rev}} = h \cdot T_{\text{rf}} \quad h \in \mathbb{N},
\]

by changing the harmonic number \( h \) of an integer value every turn. To that end, one must give the right energy gain [6] to change the revolution period of an integer number of rf period every turn.

hnj lattice design issues

If we consider the HNJ acceleration in a ring with more than two constant frequency rf cavities, all cavities must not be working at the same frequency [7, 6]. The required frequency for each cavity is a monotonic function of its position around the ring. Thus, if one wants to accelerate particles and their antiparticles simultaneously in the same ring, they must all circulate in the same direction. To achieve this, one can use a ring made of two-beam scaling FFAG cells [8].

Moreover, in the case of the acceleration of ultra-relativistic particles, the required variation of time of flight only comes from the variation of path length between each turn. It implies a necessary large average orbit excursion [6]. It is thus preferable to have low dispersion insertions (LDIs) in which rf cavities can be installed.

Example of Ring Parameters for HNJ

An example of a 3.6 to 12.6 GeV muon ring made only of two-beam scaling FFAG quadruplet cells is presented in Tab. 1. This ring comprises four LDIs in which the dispersion is reduced of about a factor two. Principle of these LDIs made of scaling type of FFAG cells is described in Ref. [6]. The maximum field is chosen below 4 T. A schematic view of the ring is shown in Fig. 1.

Simulation Tools and Results

To study lattice parameters and beam dynamics, we use a stepwise tracking code based on Runge-Kutta integration. Field distribution in the magnet mid-plane is obtained from a soft-edged geometrical field model. Field off the mid-plane is determined, satisfying the Maxwell’s equations, from a 4th order Taylor expansion.

The variation of betatron tunes, computed using this code, between 3.6 and 12.6 GeV, is found similar for both...
### Table 1: HNJ ring parameters

<table>
<thead>
<tr>
<th></th>
<th>Ring section</th>
<th>Low dispersion</th>
<th>First dispersion</th>
<th>Second dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell opening angle</td>
<td>5</td>
<td>2.5</td>
<td>4.3</td>
<td>3.2</td>
</tr>
<tr>
<td>Mean radius [m]</td>
<td>140</td>
<td>295</td>
<td>178</td>
<td>240</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>130</td>
<td>508.5</td>
<td>186.4</td>
<td>339.6</td>
</tr>
<tr>
<td>Horizontal phase adv./cell [deg.]</td>
<td>87.4</td>
<td>85.8</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Vertical phase adv./cell [deg.]</td>
<td>50.7</td>
<td>30.1</td>
<td>42.4</td>
<td>31.4</td>
</tr>
<tr>
<td>Number of these cells in the ring</td>
<td>8×4</td>
<td>8×4</td>
<td>4×4</td>
<td>4×4</td>
</tr>
</tbody>
</table>

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**Figure 1:**
- Left part: schematic view of a 3.6 to 12.6 GeV muon ring with four LDIs (regions with the largest average radius).
- Right part: Horizontal (red solid line) and vertical (dotted purple line) beta functions of 6 GeV $\mu^+$ (upper part) and $\mu^-$ (lower part) circulating in the same direction. One quarter of the ring is presented, with the LDI placed on both sides.

$\mu^+$ and $\mu^-$ circulating in the same direction, and is limited to about 0.1.

To simulate acceleration, an rf kick is given in every cell of the LDI. The sum of the rf peak voltage over one turn is 2.1 GV. Frequency of the rf cavities are chosen around 400 MHz. Before tracking, the 6D bunch of particles is prepared as follows: 1000 particles are uniformly distributed inside a transverse 4D ellipsoid (Waterbag distribution). These particles are independently distributed uniformly inside an ellipse in the longitudinal plane. Initial normalized bunch emittances, defined in a similar way than in Ref. [2], are $30 \pi$ mm.rad in both transverse planes and 150 mm in the longitudinal plane.

Tracking results, plotted in the longitudinal and transverse phase spaces are presented in Figs. 2 & 3. Similar results are found for both $\mu^+$ and $\mu^-$ beams accelerated in the same direction.

### STATIONARY BUCKET ACCELERATION

In this section we consider to use the synchrotron motion from the low energy part to the high energy part of a stationary rf bucket to accelerate particles. We examine the example of a 3.6 to 12.6 GeV muon, made of a single type of scaling FFAG cell, and presented in Tab. 2. Since all rf cavities work at the same frequency, the simultaneous acceleration of $\mu^+$ and $\mu^-$ is realized circulating these beams in opposite directions.

#### 6D Tracking Simulation of SB Acceleration

The acceleration of a 6D bunch of particles inside the stationary rf bucket of the ring described in Tab. 2 has been simulated using our tracking code. The initial particle distribution is prepared in a similar way than described in previous section. Initial normalized bunch emittances are again $30 \pi$ mm.rad in both transverse planes and 150 mm in the longitudinal plane. An rf kick that simulates the effect of a zero length rf cavity with 8 MV peak voltage is given in the middle of the long drift of every triplet cell (i.e. 1.8 GV/turn).
Table 2: Ring parameters for SB acceleration

<table>
<thead>
<tr>
<th>Lattice type</th>
<th>scaling FFAG FDF triplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inject./extrac. energy (kin.)</td>
<td>3.6/12.6 GeV</td>
</tr>
<tr>
<td>rf frequency</td>
<td>200 MHz</td>
</tr>
<tr>
<td>Synchronous energy (kin.)</td>
<td>8.04 GeV</td>
</tr>
<tr>
<td>Mean radius</td>
<td>$\sim 161$ m</td>
</tr>
<tr>
<td>Harmonic number $h$</td>
<td>675</td>
</tr>
<tr>
<td>Number of cells</td>
<td>225</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>1390</td>
</tr>
<tr>
<td>Peak rf voltage (per turn)</td>
<td>1.8 GV</td>
</tr>
<tr>
<td>$B_{\text{max}}$ (at 12.6 GeV)</td>
<td>3.9 T</td>
</tr>
<tr>
<td>Drift length</td>
<td>$\sim 1.5$ m</td>
</tr>
<tr>
<td>Horizontal phase adv./cell</td>
<td>85.86 deg.</td>
</tr>
<tr>
<td>Vertical phase adv./cell</td>
<td>33.81 deg.</td>
</tr>
<tr>
<td>Excursion</td>
<td>14.3 cm</td>
</tr>
</tbody>
</table>

Simulation results are presented in Figs. 5 & 6. No significant emittance blow-up is observed in neither the longitudinal nor the transverse phase spaces.

Figure 4: Left part: schematic view of a 3.6 to 12.6 GeV muon ring, made of 225 identical scaling FFAG cell, design for SB acceleration. Right part: the corresponding horizontal (red solid line) and vertical (dotted purple line) beta functions.

Figure 5: 6-turn acceleration cycle plotted in the longitudinal phase space. Hamiltonian contours are superimposed.

Figure 6: Initial (blue squares) and final (red dots) particles positions plot in the horizontal (left) and vertical (right) phase spaces.

SUMMARY AND DISCUSSIONS

As presented in this paper, scaling FFAGs can be used with constant rf frequency of the order of 200 MHz. It can although achieve large 6D acceptance, and thus be used from lower energies than non-scaling FFAGs.

Considering the possibility of HNJ acceleration, we developed a lattice which has the unique features of, (i) being made of different types of scaling FFAG cells, (ii) including low dispersion insertions, while (iii) allowing $\mu^+$ and $\mu^-$ beams to be accelerated in the same direction.

The SB scheme presents, on its side, several advantages. It has a significantly more efficient use of the rf than RLAs. Although less compact than non-scaling FFAG rings, the circumference of the ring presented in this paper is about half of the total length (arcs + linac) of the RLA proposed for the same energy range [2]. The simplicity of this scheme is also an advantage. A single type of cell is actually used all around the ring, which could tend to simplify the production of the magnets, and thus limit their cost.

REFERENCES