**LATTICE DESIGN FOR PROJECT-X CW SUPERCONDUCTING LINAC**

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**Abstract**

In this paper, we discuss the beam dynamics optimization of a proposed continuous wave (CW) superconducting linac for Project-X. This 3.0 GeV linac has a peak current of 5-10 mA. The average current is 1 mA, yielding an average beam power of 3 MW. The linac consists of a low energy 325 MHz section (2.5-470 MeV) containing three families of SC single-spoke resonators, an intermediate energy section (470 MeV-2.0 GeV) containing two families ($\beta_y = 0.61$ and $\beta_y = 0.9$) of 650 MHz squeezed SC elliptical cavities and finally, a high energy section (2.0-3.0 GeV) based on 1.3GHz ILC-type ($\beta_y = 1.0$) cavities. Transverse and longitudinal dynamics in the CW linac are modeled assuming the maximum 10 mA peak current. Different options for transverse focusing are considered.

**INTRODUCTION**

In May 2008, the Particle Physics Project Prioritization Panel (P5) recommended an R&D program to design a multi-megawatt proton source at Fermilab. This proton source, known as “Project X”, would open a path to discovery in neutrino science and in precision experiments with charged leptons and quarks. The initial concept for Project X – known as IC-1 – consisted of a pulsed 8 GeV linac capable of delivering up to $1.6 \times 10^{14}$ protons to the Recycler in 1.25 ms long pulses at a 2.5 Hz rate. Studies of IC-1 have identified problems with slow beam extraction and also pointed to a lack of flexibility. The IC-2 design, which is the object of this paper, addresses these issues by replacing slow extracted beam from an accumulation ring at 8 GeV with beam accelerated in a continuous wave 10 mA (or 5mA) peak, 1 mA average, linac.

**DESIGN PRINCIPLES**

From the standpoint of lattice design, much of the complexity of a proton (or ion) linac hinges on the fact that particle velocity varies during acceleration. The resulting phase slippage not only affects acceleration efficiency in multi-cell cavities but also couples transverse and longitudinal dynamics through transverse defocusing in cavities due to unequal radial kicks experienced while going through entrance and exit edge fields.

Keeping in mind that rapid change in beam size also leads to rapidly changing space charge forces, an overall objective for the optical design is to support a beam envelope with a smoothly varying amplitude: this can be accomplished by making the lattice optically quasi-periodic. A linac can be viewed as a structure with longitudinally varying (longitudinal and transverse) focusing. A well-known result (based on WKB theory) is that the equation describing the oscillation in each phase plane

$$\frac{d^2 y}{ds^2} = -k^2(s)y(s)$$  \hspace{1cm} (1)

has oscillatory solutions exhibiting a slow amplitude variation

$$y \simeq \left( \frac{k(0)}{k(s)} \right)^{1/2} A \sin \Psi(s)$$  \hspace{1cm} (2)

where

$$\Psi(s) = \int_{s_0}^s k(s)ds \text{ provided that } \left| \frac{d^2 \Psi(s)}{ds^2} \right| \simeq |k'|/|k|$$  \hspace{1cm} (3)

i.e., provided the change in oscillation period remains small with respect to the period itself. In a real machine, $k(s)$ exhibits fast local variations that result in amplitude modulation, but the general conclusion remains valid. Provided $k$ is interpreted in the context of the smooth approximation, i.e. $k(s)$ is taken to represent a suitably averaged quantity over a betatron period, ensuring the condition (3) is met will result in a smooth, slowly varying envelope.

The objective, then, is to ensure that the wavenumbers $k_x$, $k_y$, and $k_z$, interpreted as the (quasi periodic) phase advance per period length, vary as smoothly and monotonically as possible along the linac. Note that, as long as focusing perturbations, including defocusing due to space charge or rf phasing errors remain small in the sense of condition (3), the envelope is guaranteed to remain smooth.

Experience and past studies have led to a set of rules [1] that ensure minimum emittance growth in high intensity proton linacs. Although the proposed IC-2 CW linac will not operate in a space-charge dominated regime, these rules provide a sound basis for design. We simply enumerate them here: (1) The transverse phase advance per period at zero current should be set to less than 90 degrees. This ensures that space charge defocusing will not push the phase advance too much beyond 90 degrees, a region in the periodic stability diagram where the available area begins to shrink. (2) The variation of the wave numbers for both transverse and longitudinal oscillations should be adiabatic along the linac. Concretely this implies adiabatic variation both in the strength of transverse focusing elements and in the average acceleration gradient (“real estate gradient”). (3) Special attention must be paid to matching between frequency transition regions as this is the most likely source of non-adiabatic variation in both longitudinal and transverse focusing. (4) Since space charge effects naturally lead to equipartition, betatron and synchrotron oscillation temperatures should be similar (i.e. expressed in the...
same units, the emittances should roughly be equal). (5) Synchro-betatron parametric resonances (arising from the coupling between transverse rf defocusing strength and the longitudinal phase $\phi$) should be avoided. These resonances occur for $k_{\perp} = \frac{n}{2} k ||$ and generally, only the $n = 1$ resonance is important. In practice, this can be realized by making $k_T = \alpha k_{\perp}$ with $0.6 < \alpha < 0.9$.

**LAYOUT AND SEGMENTATION**

As illustrated schematically in Fig. 1, the IC-2 linac is divided into six sections. In the first, 325 MHz section, acceleration is provided by three families of single spoke cavities while transverse focusing is provided by superconducting solenoids. In the next two 650 MHz sections, acceleration is provided by two families of 5-cell elliptical cavities; transverse focusing is provided by quadrupole doublets. In the final 1300 MHz section, acceleration is provided by 9-cell ILC type elliptical cavities and focusing is provided by quadrupoles in a FODO arrangement. To improve reliability and minimize costs, all regular elements are located inside cryomodules. In the 325 MHz sections, all cryomodules are separated by short warm interconnections. In the 650 and 1300 MHz sections, cryomodules are organized in cryogenic strings defined as a set of cryomodules (typically 8) with cold interconnections, part of a common cryogenic circuit. Cryogenic strings are, in turn, separated by warm interconnections. Details of the segmentation are presented in Fig. 2.

**RESULTS**

To design the optics, we initially resorted to a combination of TRACE3D [2] and TRACK [3]. In the TRACE3D model, cavities were modeled as simple rf gaps. Since longitudinal dynamics strongly affects transverse focusing, the longitudinal optics is established first, with the constraint that the surface magnetic field in each cavity should not exceed a safe limit. For each section, beam parameters for a periodic solution were determined. Matching between sections was then performed to obtain an overall quasi-periodic solution with minimum change in phase advance per cell length. Once a satisfactory set of cavity gradients and synchronous phases was obtained, a similar procedure was employed to establish the transverse optics. Since our TRACE3D model was a bit too simplified, we resorted to TRACK, a particle tracking code which integrates accurately through cavities and focusing elements represented by 3D field maps. Many iterations between TRACE3D and TRACK proved necessary. The process is time-consuming and involves translating input files between two codes that use different conventions. Although we reached a satisfactory design, it proved difficult to improve the quality of the matching in the transitions regions because of the limitations of the matching capabilities available in TRACE3D. A bit of research led us to TRACEWIN [4], a code that has significantly better matching abilities. Specifically, TRACE3D can only deal with deterministic matching (number of parameters equal to the number of constraints), making it impossible to use many elements on either side of a transition region. Among other things, TRACEWIN can perform a type of matching that minimizes the variation of the phase advance per cell length over a region encompassing a user-specified number of cells. The algorithm involves minimizing a quadratic form involving, for each plane, a finite-difference approximation of the quasi-periodic condition (3):

$$d_2(\sigma) = \frac{\sigma_{i+1} - \sigma_{i-1} - 2\sigma_i}{\sigma_i}$$  \hspace{1cm} (4)

where $\sigma_i$ represents, for any given plane, the smooth phase advance at location $i$. The envelopes corresponding to the final result obtained after optimization using the smoothing facility in TRACEWIN are shown in Fig. 3; the corresponding wavenumber profiles are shown in Fig. 4. Note that residual discontinuities due to non-ideal matching between

Figure 1: IC2 Linac High Level Layout.

Figure 2: IC-2 Linac segmentation and period topology. In the above $R$: resonator, $D$: doublet, $S$: solenoid, $Q_F$, $Q_D$: focusing and defocusing quadrupole.

<table>
<thead>
<tr>
<th>SSR0</th>
<th>SSR1</th>
<th>SSR2</th>
<th>ILC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cryomodule</td>
<td>2 cryomodules</td>
<td>4 cryomodules</td>
<td>1 string, 21 periods</td>
</tr>
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</table>

SSR0 : 1 cryomodule, 26 periods, topology: $S-R$ ($\beta_g = 0.11$)

SSR1: 2 cryomodules, 18 periods, topology: $S-R$ ($\beta_g = 0.22$)

SSR2: 4 cryomodules, 22 periods, topology: $S-R^2$ ($\beta_g = 0.4$)

$\beta = 0.61$: 7 cryomodules, 1 string, 21 periods topology: $R-D-R$

$\beta = 0.9$: 12 cryomodules, 2 strings, 12 periods topology: $R^4-D-R^4$

ILC: 7 cryomodules, 1 string, 3.5 periods topology: $R^4-Q_F^- R^8-Q_D$ ($\beta_g = 1$)
tween sections are very small, in all three planes. Note also that the ratio $k_{x,y}/k_s$ is safely away from 2 (the $n = 1$ synchro-betatron resonance).

In the current, preferred, IC-2 lattice, transverse focusing in the 325 MHz sections is provided by superconducting solenoids. An advantage of using solenoids is that their radial focusing that can counteract exactly radial defocusing from the cavities. This is especially useful at low energy, where space charge perturbations induced by local deviations in the beam aspect ratio can be problematic. That said, we looked into the possibility of using doublet focusing in the 325 MHz sections. Our results indicate that a satisfactory solution can be obtained by using doublets at locations currently occupied by solenoids without changing cryostat dimensions or the segmentation scheme. No final conclusion has been reached; we are still considering this option, which may have some cost and operational advantages. The idealized optics shown in Figure does not make provisions for warm regions to accommodate instrumentation. A detailed design for such regions requires information that is not available at this point. Nevertheless, using TRACEWIN, we investigated the impact of introducing warm longitudinal openings at four different locations, which have warm interconnections: (1) the transition between the 325 MHz and the low energy ($\beta = 0.6$) 650 MHz sections (2) the interconnection point between the two cryogenic strings in the middle of the ($\beta = 0.6$) section (3) the transitions between the low ($\beta = 0.6$) and high energy ($\beta = 0.9$) 650 MHz sections (4) the transition between the ($\beta = 0.9$) 650 MHz and the ($\beta = 1$) 1300 MHz ILC section. At locations (3) and (4), it is straightforward to insert a 12.6 m warm region with a transverse focusing element at its center respectively mimicking a period or a half-period. Since rf defocusing is small at high energy, a minimum amount of matching is required. At location (2), it is possible to introduce a 2 m space and rematch by modifying the synchronous phase of the cavities and the strength of focusing elements in the close vicinity. Not unexpectedly, the most problematic location for opening up longitudinal space is (1), corresponding the the lowest energy. In that case, a solution was obtained by opening up 2 m of space (Fig. 5). An additional warm quadrupole is added in the center of that opening, to facilitate matching which otherwise proved very difficult because this transition involves a change in the transverse focusing scheme, from radially focusing solenoids to doublets. At this point it appears that opening up more than 2 m of space at (1) and (2) may be possible, but doing may involve a more elaborate constrained matching procedure.

**CONCLUSION**

We have produced a realistic conceptual design for the CW IC2 Project-X linac optics. Many issues, including the impact of misalignments and rf phase and amplitude errors remain to be studied in details. Recently, intra-beam stripping has been identified as a mechanism for of particle loss which might be of significance for the IC-2 linac (which accelerates $\text{H}^-$ ions). Mitigating the losses induced by this mechanism may require modifications to the optics.

**REFERENCES**