CALCULATION OF SECOND ORDER MOMENTS FOR ION BEAM IN DEGRADER

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Abstract

In some experiments one needs to run an ion beam through a thin metal foil (degrader) in order to decrease the ion energy on value of some MeV/amu. One can calculate the final ion energy, angular and energy stragglings, which the beam attains in the degrader, with the help of, for example, codes LISE++ [1] or TRIM [2]. The formulae for calculation of new second order moments such as $\overline{x^2}$, $\overline{x'^2}$, $\overline{xx'}$, $\overline{y^2}$, $\overline{y'^2}$, and $\overline{yy'}$, were obtained. They give a possibility to calculate new Twiss parameters of the beam. The formulae for calculation of new final beam momentum spread $\overline{(\Delta p/p)^2}$, new emittances $\varepsilon_{x,y}$ values, new values of the dispersion functions (D_x, D_y) and their derivatives were also obtained. These new ion beam parameters allow one to calculate further beam motion along the channel on the other side of the degrader.

INTRODUCTION

In some experiments a task arises to decrease the ion beam energy from the value W_0 to the value W with the help of a thin (for example, tantalum) foil placed in a channel point having the coordinate z. Then it is necessary to determine the new beam parameters for calculation its further tracing behind the degrader since the initial beam parameters suddenly change when the beam passes the degrader. The formulae for carrying out such recalculations are obtained in this work.

SECOND ORDER MOMENTS

It was assumed that the following beam parameters just before the metal foil are known:

1) The values of the horizontal D_{x0} and vertical D_{y0} dispersion functions and its derivatives with respect to the longitudinal coordinate D'_{x0}, ; D'_{y0};

2) The value of the initial ion RMS momentum spread δ_0;

3) The ion energy W_0;

4) The spatial and angular second order moments $\overline{x_0^2}$, $\overline{x_0'^2}$, $\overline{xx'}_0$, $\overline{y_0^2}$, $\overline{y_0'^2}$, and $\overline{yy'_0}$;

5) The transverse beam emittances ε_{x0} and ε_{y0};

For ion beams, extracted from a cyclotron, $D_{y0} << D_{x0}$ and $D'_{y0} << D'_{x0}$. Therefore one considers in calculations that $D_{y0} = 0$ and $D'_{y0} = 0$.

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For a given foil thickness one defines with the help of, for example, code LIZE++ [1] or TRIM [2] the following beam parameters after its passing the degrader:

1) The final ion beam energy W;

2) RMS ion energy straggling ΔW defining the value of additional RMS ion momentum spread δ_S:

$$\delta_S^2 = \frac{1}{4}\cdot\overline{\left(\frac{\Delta W}{W}\right)^2} \tag{1}$$

3) The additional ion RMS angle spread δ_θ.

The findings allow one to calculate new parameters of the ion beam, having passed through the foil, in assumption that there are no any correlations between the initial particle coordinates, angles and their changes in the degrader. The transversal beam dimensions don't change when it passes the foil, i.e. the following conditions are fulfilled:

$$\overline{x^2} = \overline{x_0^2} \quad ; \quad \overline{y^2} = \overline{y_0^2} \tag{2}$$

The rest of second order moments are calculated as:

$$\overline{x'^2} = \frac{W_0}{W}\cdot\overline{x_0'^2} + \frac{\delta_\theta^2}{2} \;;\; \overline{y'^2} = \frac{W_0}{W}\cdot\overline{y_0'^2} + \frac{\delta_\theta^2}{2} \tag{3}$$

as well as

$$\overline{xx'} = \sqrt{\frac{W_0}{W}}\cdot\overline{xx'_0} \;;\; \overline{yy'} = \sqrt{\frac{W_0}{W}}\cdot\overline{yy'_0} \tag{4}$$

The formulae (3) were checked up with the help of the code TRIM. The ensemble of ^{20}Ne ions (their number was 10^4) having the energy 20 MeV/amu and zero angle spread was run through the carbon foil having the thickness of 200 μm. The calculated distribution of particle velocities on the phase plane {x′, y′} just behind the degrader is shown in Fig. 1

It was proved that the following equalities:

$$\overline{x'^2} = \frac{\delta_\theta^2}{2} \;;\; \overline{y'^2} = \frac{\delta_\theta^2}{2} \tag{5}$$

fulfil with accuracy 1%.

The final beam momentum spread taking into account influence of the foil can be found as:

$$\overline{(\Delta p/p)^2} = \delta^2 + \delta_S^2 \tag{6}$$

where

$$\delta^2 = \frac{W_0}{W}\cdot\delta_0^2 \tag{7}$$

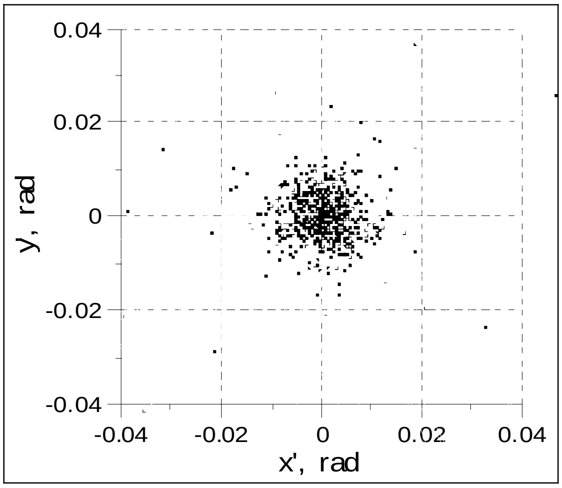

Figure 1. Distribution of ion velocities

DISPERSION FUNCTIONS

The values of the dispersion functions and their derivatives can be found by formulae:

$$D_x = \frac{\overline{x\Delta p/p}}{\overline{(\Delta p/p)^2}} = \sqrt{\frac{W}{W_0}}\cdot\frac{D_{x0}}{1+\delta_S^2/\delta^2}$$
$$D'_x = \frac{\overline{x'\Delta p/p}}{\overline{(\Delta p/p)^2}} = \frac{D'_{x0}}{1+\delta_S^2/\delta^2} \tag{8}$$

BEAM EMITTANCE

The horizontal RMS beam emittance ε_x is defined by formula taking into account the non-zero momentum spread of the beam:

$$\varepsilon_x^2 = \left(\overline{x^2} - D_x^2\overline{(\Delta p/p)^2}\right)\left(\overline{x'^2} - D_x'^2\overline{(\Delta p/p)^2}\right) -$$
$$-\left(\overline{xx'} - D_x D'_x\overline{(\Delta p/p)^2}\right)^2 \tag{9}$$

Thereby behind the degrader the following expression is valid:

$$\varepsilon_x^2 = \varepsilon_{x0}^2\frac{W_0}{W} + \frac{\delta_\theta^2}{2}\cdot\left(\overline{x_0^2} + \frac{W}{W_0}D_{x0}^2\frac{\delta_S^2}{1+\delta_S^2/\delta^2}\right) +$$
$$+\left(\overline{x_0'^2}\,D_{x0}^2 - 2\overline{xx_0'}D_{x0}D'_{x0} + \overline{x_0^2}D_{x0}'^2\right)\frac{\delta_S^2}{1+\delta_S^2/\delta^2} \tag{10}$$

The vertical RMS beam emittance ε_y is defined by usual manner and for its value after degrader we have the formula:

$$\varepsilon_y^2 = \varepsilon_{y0}^2\cdot\frac{W_0}{W} + \frac{\delta_\theta^2}{2}\overline{y_0^2} \tag{11}$$

TWISS FUNCTIONS

The new values of the Twiss functions $\beta_{x,y}$ and $\alpha_{x,y}$ can be found in the following way

$$\beta_x = \frac{\overline{x^2} - D_x^2\,\overline{(\Delta p/p)^2}}{\varepsilon_x} = \beta_{x0}\frac{\varepsilon_{x0}}{\varepsilon_x}$$
$$\beta_y = \frac{\overline{y^2}}{\varepsilon_y} = \beta_{y0}\frac{\varepsilon_{y0}}{\varepsilon_y} \tag{12}$$

$$\alpha_x = -\frac{\overline{xx'} - D_x D'_x\overline{(\Delta p/p)^2}}{\varepsilon_x} =$$
$$= \sqrt{\frac{W_0}{W}}\frac{\varepsilon_{x0}}{\varepsilon_x}\left(\alpha_{x0} - \frac{W_0}{W}\frac{D_{x0}D'_{x0}}{\varepsilon_{x0}}\frac{\delta_S^2}{1+\delta_S^2/\delta^2}\right) \tag{13}$$
$$\alpha_y = -\frac{\overline{yy'}}{\varepsilon_y} = \sqrt{\frac{W_0}{W}}\frac{\varepsilon_{y0}}{\varepsilon_y}\alpha_{y0}$$

ARGON BEAM TRANSPORT SIMULATION

The change of the ion beam half-dimensions calculated with the help of COSY INFINITY [3] code for the $^{40}Ar^{17+}$ ion beam extracted from the cyclotron MC-400 is shown in Fig. 2.

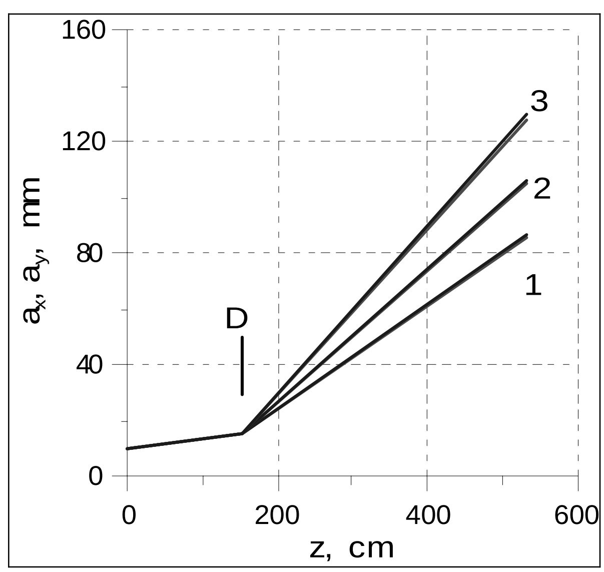

Figure 2. The curves 1, 2 and 3 correspond to the final ion energy 6 MeV/amu, 4.5 MeV/amu and 3 MeV/amu

The initial beam energy W_0 = 7.8 MeV/amu. It was considered that all magnetic fields were switched off in the channel. The beam passes through the degrader D which contains tantalum foil having the diameter 90 mm. With the help of this foil the beam energy decreases down to three different values: W = 6 MeV/amu (δ_{Ta} = 7.2 μm), W = 4.5 MeV/amu (δ_{Ta} = 12.6 μm) and 3 MeV/amu (δ_{Ta} = 17.6 μm). As one can see from Fig. 2, the ion beam sharply enlarges behind the degrader in all three cases. It is connected mainly with the considerable angle straggling which the beam gets when passing the degrader.

CONCLUSION

The formulae obtained in this work allows one to define the new ion beam parameters after its passing the degrader and calculate its further motion along the channel.

REFERENCES

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