 Vernier Scan Results from the First RHIC Proton Run at 250 GeV *

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Abstract

Using the Vernier scan or Van der Meer scan technique, where one beam is swept stepwise transversely across the other while measuring the instantaneous luminosity as a function of beam displacement, the transverse beam profiles, the luminosity and the effective cross section of the detector in question can be measured. This report briefly recalls the vernier scan technique and presents results from the first RHIC polarized proton run at 250 GeV per beam in 2009.

Introduction

The event or collision rate \( \dot{N} \) for a given process of cross section \( \sigma \) produced in a machine running with luminosity \( \mathcal{L} \) is given by:

\[
\dot{N} = \mathcal{L} \sigma
\]  

The cross section observed by the experiments is essential to absolutely normalize the experimental data in proton collisions. If the cross section for a process is not well known one way of determining the cross section is to measure the instantaneous luminosity. At the Relativistic Heavy Ion Collider (RHIC) collisions are observed in two experimental areas, STAR and PHENIX, where the collision rates are measured with the ZDCs (Zero Degree Calorimeter) and, in the case of PHENIX, with BBCs (Beam Beam Counter). While the two ZDCs are mostly identical, the BBCs are of different shape and acceptance. They provide independent measurements which can help understanding the systematics. They all consist of two identical parts on each side of the IP and provide coincidence rates. In this report, we describe how the Vernier scan method was used to measure the effective cross section of the ZDC monitors for proton-proton collisions at 250 GeV.

The Vernier Scan Method

The Vernier scan method for luminosity determination was pioneered by Van Der Meer at the ISR [1]. The transverse size and shape of the beam overlap region is measured by recording the relative interaction rates as a function of the transverse beam separation \( \delta \). For gaussian beams we get:

\[
\mathcal{L}(\delta u) = \mathcal{L}_0 \exp \left[ -\frac{\delta u^2}{2(\sigma_{u1}^2 + \sigma_{u2}^2)} \right],
\]

where

\[
\mathcal{L}_0 = \frac{N_1 N_2 \frac{f}{\sqrt{2\pi \sigma_{x\text{eff}} \sigma_{y\text{eff}}}} f N_b}{2\pi \sqrt{(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{y1}^2 + \sigma_{y2}^2)}}
\]

where \( u = x, y, N_1 \) and \( N_2 \) are the bunch intensities, \( N_b \) the number of colliding bunches and \( f \) the revolution frequency. A fit of the measured interaction rates as function of the separation will allow to determine the effective beam size as well as the maximum achievable collision rate. In reality additional effects such as crossing angle or hourglass effect [2] require correction factors to be applied to the absolute luminosity and will have to be taken into account when computing the systematic error.

Beam Profile

The definition of the luminosity presented above assumes perfectly Gaussian beams. In hadron machines, where the damping is very low, this is not always the case and non-Gaussian tails can appear. These non-Gaussian components of the beam still contribute to the overall luminosity and have to be taken into account while computing the overlap integral. The core of the beam, which very often remains Gaussian, is the main contributor to the luminosity. A convenient way to include the tails in the model is to fit the profile with a double Gaussian. The luminosity as function of the separation is then:

\[
\mathcal{L}(\delta x, \delta y) = \frac{N_1 N_2 f N_b}{A_{\text{eff}}} F(\delta x, \delta y)
\]

where \( A_{\text{eff}} \) is the effective area and is defined as:

\[
A_{\text{eff}} = \frac{\int_{-\infty}^{+\infty} F(\delta x, 0) d\delta x \int_{-\infty}^{+\infty} F(0, \delta y) d\delta y}{F(0, 0)}
\]

\( F(\delta x, \delta y) \) is the function describing the overlap profile. For a double Gaussian we have:

\[
F_u(\delta u) = A_{1u} \exp \left[ -\frac{\delta u^2}{2\sigma_{1u}^2} \right] + A_{2u} \exp \left[ -\frac{\delta u^2}{2\sigma_{2u}^2} \right]
\]

This leads to

\[
\mathcal{L}_0 = \frac{N_1 N_2 f N_b}{2\pi \sigma_{x\text{eff}} \sigma_{y\text{eff}}} A_{1u} A_{2u}
\]

with

\[
\sigma_{x\text{eff}} = \frac{A_{1u} \sigma_{1u} + A_{2u} \sigma_{2u}}{A_{1u} + A_{2u}}
\]

Figure 1 shows the comparison between a Gaussian + constant fit and a double Gaussian fit. Looking at \( \chi^2/\text{ndf} \) it is clear that the double Gaussian fit is more suited to the beam profile. At RHIC this effect was first and systematically seen during the 2009 proton run.

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CROSSING ANGLE AND HOURGLASS EFFECT

When $\beta^*$ is equal or smaller than the longitudinal beam size the so-called hourglass effect introduces an $s$-dependence on the beam sizes. For round Gaussian beams ($\beta^*_x = \beta^*_y$) the luminosity is [2]:

$$L_{HG} = L_0 \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-t^2}}{1 + \frac{t^2}{\tau^2}} dt = L_0 \sqrt{\pi} t e^{t^2} \text{erfc}(t)$$

where $\text{erfc}(z)$ is the complementary error function, $t^2 = \frac{\beta^*}{\sigma_{L_1}^2 + \sigma_{L_2}^2}$ and $t = \frac{\sqrt{r^2}}{\sigma_{L_1}^2 + \sigma_{L_2}^2}$. During the 2009 proton run $\beta^*$ was equal to 0.7 m and $\sigma_s$ about 1 m, the hourglass effect was therefore non-negligible. The PHENIX BBC is situated approximately at ±0.75 m from the IP with a vertex cut applied at ±0.3 m and the ZDC at ±20 m. The ZDC will therefore see the full hourglass effect while the BBC practically measures the beam size at the IP. Comparing the two would then give a measurement of the hourglass effect. The luminosity as a function of the separation in the presence of hourglass effect is expressed as:

$$L(\delta x, \delta y) = L_0 \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-t^2}}{1 + \frac{t^2}{\tau^2}} F(\delta x, \delta y, t) dt$$

where $F(\delta x, \delta y, t) = F_x(\delta x, t) F_y(\delta y, t)$ and

$$F_u(t) = \exp \left[ -\frac{\delta u^2}{2(\sigma_{L_1}^2 + \sigma_{L_2}^2)} \frac{1}{1 + \frac{t^2}{\tau^2}} \right]$$

The effective beam size measured by the scan can be calculated using Equation 5:

$$\sigma_{\text{eff}} = \sqrt{\sigma_{1u}^2 + \sigma_{2u}^2} \frac{1}{\sqrt{\pi}} \frac{e^{-t^2}}{1 + \frac{t^2}{\tau^2}} K_0 \left[ t^2 \right]$$

where $K_0$ is the modified Bessel function. We can see that a correction has to be applied in order to compute the luminosity from Equation 9.

Figure 1: Profile as measured from the ZDC in PHENIX. A Gaussian fit is shown in blue and a double Gaussian fit with fit parameters is shown in red.

Figure 2: Ratio of the ZDC and BBC beam sizes as measured from the Vernier scans. $\sigma_s$ represents the convoluted bunch length.

Figure 2 shows that the measurements agree reasonably well with the model which assumes no crossing angle and perfect Gaussian beams which could explain the small discrepancies.

Figure 3: Reduction factor due to the combined effect of the crossing angle and the hourglass effect for $\sigma_s = 1$ m and $\beta^* = 0.7$ m.

The crossing angle introduces an additional reduction factor to the luminosity. Using the method presented in [4] and the model illustrated in Figure 3 a crossing angle uncertainty of 0.05 mrad was computed and used to derive a systematic error of 1%.

BEAM-BEAM DEFLECTION

When two bunches cross each other the trajectories of the particles are modified by a horizontal and vertical angle $\Delta x'$ and $\Delta y'$ due to the electromagnetic field of the counter rotating bunch. In the case of equal round Gaussian beams the radial deflection [3] can be expressed as

$$\Delta r = -\frac{8\pi \xi \sigma^2}{\beta^*} \frac{1}{r} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right) \right];$$

where $r^2 = x^2 + y^2$, $\beta^*$ is $\beta$-function at the IP and $\xi$ is the linear beam-beam parameter defined as

$$\xi = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2},$$

where $N$ is the number of bunches.
an angular kick $\theta$ at a position $s_0$ can be translated into an orbit change at a given position $s$:

$$y(s) = \frac{\sqrt{\beta(s)\beta(s_0)}\theta}{2 \sin \pi \nu} \cos (\pi \nu - |\varphi(s) - \varphi(s_0)|) \quad (15)$$

where $\nu$ is the betatron tune and $\varphi$ the phase. This formula was used to compute the beam-beam deflection kick at the IP from the orbit changes at the Dx BPMs left and right. Computing the beam-beam deflection angle allows for a measurement of the effective beam size and the beam-beam parameter.

Figure 4: Beam-beam deflection scan in the horizontal plane observed at STAR.

Figure 4 illustrates the evolution of the beam-beam deflection angle as a function of the separation. This effect was systematically observed but most of the time the BPM resolution was not sufficient and the measurement was dominated by the error bars. Only the fits with reasonable $\chi^2$ and error bars were taken into account.

Figure 5: Beam-beam deflection scan results in the horizontal plane observed at STAR and PHENIX.

Figure 5 compares the fitted beam-beam parameters with the ones calculated from the Vernier scan results assuming nominal $\beta^*$. It shows very good agreement between measurements and expected values. The fitted beam sizes had error bars of 10% to 20% but were generally in good agreement with what was measured from the Vernier scans.

**RESULTS**

The systematics and data analysis were presented in [4] and [5]. The same approach was used for the 2009 proton run.

The difference between the cross sections in STAR and PHENIX can be explained by the slightly different detector configurations in the two experiments.

**CONCLUSION**

Vernier scans were performed during 250 GeV polarized proton run in PHENIX and STAR. The measured beam profile appeared to be not perfectly Gaussian but could be fitted with a double Gauss which still gives an analytical formulation of the luminosity. A measurement of the hourglass effect was done in PHENIX and a beam-beam deflection angle was observed. Both measurements agree with the expected values. Effective cross sections were derived for the ZDCs in PHENIX and STAR.

**REFERENCES**