Simulations of Plasma Wakefield Acceleration at FACET and Beyond

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Plasma simulation has greatly impacted on PBA research.
Why Plasma?

Gauss’ Law

\[ \nabla \cdot E \sim ik_p E = -4\pi en_1 \]

\[ k_p = \omega_p / V_{ph} \approx \omega_p / c \]

\[ n_1 \sim n_o \]

\[ \Rightarrow eE \sim 4\pi en_o e^2 c / \omega_p = mc\omega_p \]

or \[ eE \sim \sqrt{\frac{n_o}{10^{16} \text{cm}^{-3}}} \text{10 GeV/m} \]

\(~1000 \text{ times larger than the conventional accelerators}\)
How to Make a Plasma Wake Field?

Nonlinear Process

LWFA*: Wake: phase velocity = driver velocity (V_g or V Beam)

PWFA*: Blow-out Regime[1,2]

LWFA: Tajima and Dawson 1979
PWFA: Chen, Dawson et al., 1985

How to Simulate Plasma Based Accelerator?

Particle-In-Cell Simulation

\[ \frac{d\vec{p}}{dt} = \frac{q}{m} (\vec{E} + \frac{\vec{p}}{\gamma} \times \vec{B}) \]

Computational cycle

- Particle positions and velocities update
- Interpolation for EM field at particle positions
- Charge and current deposition at grids
- Electric and magnetic field solve

Massively Parallel Simulation Code

\[
\begin{align*}
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J} \\
\n\nabla \cdot \vec{E} &= \rho \\
\n\nabla \cdot \vec{B} &= 0
\end{align*}
\]

Simulation of PBA

Beam Particles: $10^{10}$

Plasma Length: $\sim 1 \text{ m}$

Moving Window

Plasma Particles: $10^{12}$

Maxwell's Eqns

\[
\begin{align*}
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J} \\
\n\nabla \cdot \vec{E} &= \rho \\
\n\nabla \cdot \vec{B} &= 0
\end{align*}
\]

All particles move self-consistently

~ 500 um
QuickPIC simulation of two-bunch electron-driven PWFA.

QuickPIC simulation of LWFA with a beam load.

The drive beam evolves in a much longer time scale than the plasma particles.
QuickPIC\textsuperscript{[1,2]} is a 3D parallel Quasi-Static PIC code, which is developed based on the framework UPIC\textsuperscript{[3]}.

Full PIC (Osiris):

\[ dt \sim 0.05 \omega_p^{-1} \]

Courant Condition

QS PIC (QuickPIC):

\[ dt \sim 20.0 \omega_p^{-1} \]
\[ \sim \sqrt{\gamma} \text{ of the beam} \]
\[ \sim \omega_0 / \omega_p \]

1000 Times Faster

Math Behind QuickPIC

\begin{align*}
(x, y, z; t) &
\quad \downarrow
(x, y, \xi = ct-z, s = z) \\
\partial_s << \partial_\xi &
\quad \downarrow
\text{Plasma: } (x, y; \xi) \\
\downarrow &
\text{Beam: } (x, y, \xi, s)
\end{align*}

\begin{align*}
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J} \\
\nabla \cdot \vec{E} &= \rho \\
\nabla \cdot \vec{B} &= 0
\end{align*}

\begin{align*}
\nabla_\perp \cdot \vec{E} &= \frac{\partial}{\partial \xi}(\vec{B} \cdot \hat{\xi} \times \vec{E}) \\
\nabla_\perp \times \vec{B} - \vec{J} &= \frac{\partial}{\partial \xi}(\vec{E} + \hat{z} \times \vec{B}) \\
\nabla_\perp \cdot \vec{E} - \rho &= \frac{\partial}{\partial \xi} \hat{z} \cdot \vec{E} \\
\nabla_\perp \cdot \vec{B} &= \frac{\partial}{\partial \xi} \hat{z} \cdot \vec{B}
\end{align*}

\[
\frac{\partial}{\partial z} = -\frac{\partial}{\partial \xi} + \frac{\partial}{\partial s}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \xi}
\]

*P. Sprangle, et al., PRA 41, 4463 (1990)*
Equations in QuickPIC

\[ \mathbf{E}_\perp + \mathbf{\hat{z}} \times \mathbf{B}_\perp = -\nabla_\perp \cdot \psi \]
\[ \nabla_\perp^2 \psi = -(\rho - J_z) \]
\[ \nabla_\perp^2 \mathbf{B}_\perp = \mathbf{\hat{z}} \times \left( \frac{\partial}{\partial \xi} \mathbf{J}_\perp + \nabla_\perp \cdot \mathbf{J}_z \right) \]
\[ \nabla_\perp^2 B_z = -\nabla_\perp \times \mathbf{J}_\perp \]
\[ \nabla_\perp^2 E_z = \nabla_\perp \cdot \mathbf{J}_\perp \]

\[ \frac{d \mathbf{p}_\perp}{d\xi} = \frac{q/m}{1 - v_z} \left[ \mathbf{E} + \mathbf{\bar{v}} \times \mathbf{B} \right] \]

\[ \frac{\partial}{\partial \xi} (\rho - J_z) + \nabla_\perp \cdot \mathbf{J}_\perp = 0 \]

\[ \frac{\partial}{\partial \xi} Q (1 - v_z) = 0 \]

For each plasma particle: \( Q \) varies along \( \xi \) according to its \( v_z \)


Iteration Required! Coupled with equation of motion.
Embeds a 2D PIC code inside a 3D PIC code based on UPIC Framework.
Current Status of QuickPIC

1. Improved Iteration Loop

2. Multiple Field Ionization Module

3. Beam Particle Tracking

4. Plasma Particle Tracking
Current Status of QuickPIC

Time for pushing one particle for one step using a single processor (double precision): ~770 ns
On-Going Work:
- MPI+OpenMP
- GPU Acceleration
- Python version
- Open Source Project

SKELETON CODES

Current Status of QuickPIC

Support

UPIC 2.0

*Shttp://picksc.idre.ucla.edu*
Many research papers use QuickPIC as the simulation tool.
FACET provides high-energy, high peak current e⁻ & e⁺ beams for PWFA experiments at SLAC.
Former Experiments on FFTB at SLAC demonstrated a more than 50 GeV/m accelerating gradient can be produced in PWFA over a meter long scale.

Demonstrate High Energy Transfer From a Drive Bunch to a Trailing Bunch: Design Experiment

*T. Katsouleas et al., Part. Accel (1987)

Two-Bunch e⁻ PWFA


Head Erosion For FACET BEAM

*W. An et al, 16 101301, PRSTAB (2013)

A Collider Requires Positrons

E. Adli et al., IPAC 2014
The $e^+$-Plasma Interaction Differs from the $e^-$-Plasma Interaction

First High-gradient $e^+$ PWFA

$E_{\text{gain}} > 4\text{GeV}$!
Generation of Mono-Energetic $e^+$ with High Gradient

Drive Beam: $\sigma_r = 70.0 \, \mu m$, $\sigma_z = 30.0 \, \mu m$, $N_2 = 1.4 \times 10^{10}$, $\varepsilon_N = (50,200) \, \text{mm} \cdot \text{mrad}$

Plasma Density: $8.0 \times 10^{16} \, \text{cm}^{-3}$ (1.5 meters long)

*S. Corde et. al, 524, 442 Nature(2015).*
Generation of Mono-Energetic $e^+$ with High Gradient

**Drive Beam:** $\sigma_r = 70.0 \ \mu m$, $\sigma_z = 30.0 \ \mu m$, $N = 1.6 \times 10^{10}$, $\varepsilon_N = (50,200)$ mm·mrad

**Plasma Density:** $8.0 \times 10^{16}$ cm$^{-3}$ (1.3 meters long including two 15 cm long density ramps)

'$\xi$' values:
- $\xi = -1.25$
- $\xi = 1.25$
Generation of Mono-Energetic $e^+$ with High Gradient

Drive Beam: $\sigma_r = 70.0 \, \mu m$, $\sigma_z = 30.0 \, \mu m$, $N = 1.6 \times 10^{10}$, $\varepsilon_N = (50,200) \, \text{mm-mrad}$

Plasma Density: $8.0 \times 10^{16} \, \text{cm}^{-3}$ (1.3 meters long including two 15 cm long density ramps)

The Pseudo Potential $\psi$

$\xi = 1.25$

$\xi = -1.25$
Generation of Mono-Energetic $e^+$ with High Gradient
Another Way to Accelerate Positron Plasma Hollow Channel

Kinoform

1.6 GeV Energy Gain for in 1 meter
0.2% Energy Spread (Initial E.S. is 0)
FACET-II
Science Opportunities Workshops

12-16 October, 2015
SLAC National Accelerator Laboratory
Menlo Park, CA

FACET-II is a new user facility that will provide unique capabilities to develop advanced acceleration and coherent radiation techniques with high-energy electron and positron beams. FACET-II provides a major upgrade over current FACET capabilities and the breadth of the potential research program will make it truly unique.

Even High Efficiency and High Quality Beam Aiming to the Future Linear Collider.
Beam Loading Scenarios & Ion Motion
• Theory allows for designing highly efficient stages that maintain excellent beam quality.
• Simulation for PWFA-LC showed ~ 50% energy transfer efficiency with <1% energy spread
• BUT.......
Matched Beams Lead to Ion Collapse that Degrades Emittance

Trailing beam density:

\[ n_b = \frac{N}{(2\pi)^{3/2}\sigma_r^2\sigma_z} \]

Efficient beam loading and high luminosity:

\[ N = 1 \times 10^{10} \]

Matching:

\[ \sigma_r^2 = \sqrt{\frac{2}{\gamma}}k_p^{-1}\epsilon_N \]

Energy spread:

\[ \sigma_z = \alpha \frac{c}{\omega_p} \quad (\Lambda > 1) \]

Leads to:

\[ \frac{n_b}{n_0} = 1.4 \times 10^4 \frac{N}{1 \times 10^{10}} \frac{\mu m - rad}{\sqrt{\epsilon_{Nz}\epsilon_{Ny}}} \sqrt{\frac{\text{Energy}}{250 GeV}} \frac{1}{\alpha} \]

For collider parameters:

\[ \frac{n_b}{n_0} \approx 10^4 - 10^5 \]

Ion motion, which can degrade the accelerating and focusing fields, occurs when \( n_b/n_0 \sim M/m \)
Ions collapse!

\[ \frac{n_b}{n_p} \gg \frac{m_{ion}}{m_e} \Rightarrow \Delta \phi \gg 1 \]

Big Challenge

Plasma and Beam Densities

Ion Column

400 μm x 400 μm x 300 μm Box

16384 x 16384 x 2048 Cells

12 μm x 12 μm x 60 μm Box

4096 x 4096 x 512 Cells

Δ⊥ ≈ 25 nm

Δ⊥ ≈ 3 nm
Trailing Beam: $\sigma_z = 10.0 \, \mu m$, $N = 1.0 \times 10^{10}$, $\sigma_x = 0.463 \, \mu m$, $\varepsilon_{Nx} = 2.0 \, \text{mm-mrad}$, $\sigma_y = 0.0733 \, \mu m$, $\varepsilon_{Ny} = 0.05 \, \text{mm-mrad}$

$\Upsilon = 48923.7 \, (25 \, \text{GeV})$, Plasma Density: $1.0 \times 10^{17} \, \text{cm}^{-3}$

In Li, the emittance in $x$ does not change, and in $y$ direction it only increase by 20%.

In H, the emittance in $x$ increase by 10%, and in $y$ direction it increases by 70%.
High energy gain and high efficiency acceleration of both $e^-$ and $e^+$ in the PWFA have been demonstrated in the experiments at FACET.

QuickPIC simulation results for these experiments show a good agreement with the experimental results. The simulation study also provides us more detailed information that can help us explore the unknown and guide our future experiments.