Abstract
We discuss coupled bunch and single bunch instabilities caused by electron cloud in positron circular accelerators. Unstable mode spectrum, which characterizes the instabilities, is focused.

INTRODUCTION
Electron cloud causes coupled bunch and single bunch instabilities. The coupled bunch instability caused by electron cloud has been observed at KEK-PF, since start of positron operation in 1988. The instability was confirmed at BEPC in IHEP, China. The instability has been observed in KEKB-Low Energy Ring, which is positron storage ring. Weak solenoid coils were wound along the whole ring to protect electrons near the beam. The unstable mode of the coupled bunch instability was affected by the solenoid status, ON or OFF. Corrective electron motion reflected to beam unstable mode. The coupled bunch instability has been observed at DAFNE in Frascati, Italy. DAFNE is a small $e^+e^-$ collider ring with the circumference of 98m. Bending magnets and wiggler magnets are occupied in a large part of ring. It seemed that electrons in the bending field play important role. The unstable mode was characteristic for the electron motion in the bending field.

Single bunch instability has been observed in KEKB-LER. Generally fast head-tail instability is caused by merge between 0 and -1 synchrotron sideband modes in positron ring. In the single bunch instability, clear positive side band $v_i+a_i$ (1<$a_i$<2) has been observed. The similar signal has been observed at PETRA-III, DESY, Germany. The instability has been observed in Cesr-TA, Cornell, USA. The signal appears as negative side band, $v_i-a_i$.

Simulations have been performed to explain the both of single and coupled bunch instabilities. We present the simulation results with focusing unstable mode in this paper.

COUPLED BUNCH INSTABILITY DUE TO ELECTRON CLOUD

Simulation of Electron Cloud Build Up and Coupled Bunch Instability

Beam-electron cloud system in multi-bunch regime is described by following equations:

$$\frac{d^2x_p}{ds^2} + K(s)x_p = \frac{2eN_c}{\gamma} \sum_{c=1}^{N_c} F_G(x_p - x_c) \delta_p(s - s_c)$$

(1)

$$\frac{dx_p}{dt} = \frac{e}{m_e} x_p \times B - 2N_e e_c \sum_{p=1}^{N_c} F_G(x_e - x_p) \delta_p(t - t_p(s_e)) - \frac{r_e e_c^2}{\gamma} \frac{\partial \phi(x_e)}{\partial x_e}$$

(2)

where the electric potential of electron cloud is given by

$$\Delta \phi(x) = \sum_{c=1}^{N_c} \delta(x - x_c)$$

(3)

Bunches, which are rigid Gaussian shape in transverse and are located equal spacing along s, are represented by their center of mass $x_p$. $F_G$ is expressed by the Bassetti-Erskine formula.

The electron cloud build-up is simulated by integrating the second equation for the motion of macro-electrons $x_p$ under $x_p=0$. The initial condition of electrons, where and when electrons are created, is sketched in Figure 1. When beam pass through the chamber cross-section, electrons are created with a energy distribution. When an electron hits the wall, secondary electrons are produced with a probability.

Figure 1: Electron cloud build-up model.

The coupled bunch instability is studied by solving the two coupled equations (1) and (2).

Coupled Bunch Instability in KEK-PF

Progress of electron cloud effects in these 17 years (since 1995) started from the interpretation of an instability observed in KEK-Photon Factory. Very strong coupled bunch instability had been observed since the positron storage had started. The threshold of the instability is very low 10-15mA. Either of positron or electron could be storaged by changing the polarity of the magnets. The instability is observed only in positron storage. Figure 2 shows frequency spectrum for the instability published in [1]. The first electron cloud build-up code (PEI) is developed in 1995 [2], and the coupled bunch instability was interpreted as a wake effect of the electron cloud. The wake force was estimated by perturbation of the cloud due to a passage through of a shifted bunch [2]. Figure 3 shows the simulated wake force.

Growth rate for each mode is calculated by the wake force. Figure 4 shows unstable mode spectrum estimated by the simulated wake force. The growth rate is 0.3msec for $m$=250-300. The corresponding frequency is (h-m-
\( \nu_j f_0 \), where \( \nu \) and \( f_0 \) are beam harmonic number and revolution frequency, respectively. The growth rate and mode spectrum well agreed with the measurements.

Figure 2: Unstable mode spectrum measured in KEK-PF [1].

![Figure 2](image1.png)

Figure 3: Simulated wake force due to electron cloud in KEK-PF [2].

![Figure 3](image2.png)

Figure 4: Unstable mode spectra due to the wake force in Figure 3 [2].

![Figure 4](image3.png)

Coupled Bunch Instability in BEPC

Experiments were held to study reappearance of the electron cloud instability in BEPC since 1997. Both/either of positron and/or electron beams could be stored in BEPC; where the circulating directions are opposite.

Figure 5 shows mode spectra in positron and electron storage, respectively. Multi-mode instability similar as KEK-PF observation (Figure 2) was seen in BEPC. While single mode instability in electron storage also agreed with KEK-PF observation.

Simulation using Eqs.(1) and (2) gives beam centroid motion. FFT of the centroid positions gives unstable mode spectrum, which is compared with measurement. The spectra well agree with the measurements as shown in Figure 6.

![Figure 5](image4.png)

![Figure 6](image5.png)

Coupled Bunch Instability in KEKB-LER

Coupled bunch instability due to electron cloud has been observed since the early stage of the commissioning of KEKB. For narrow bunch spacing 4-6 ns, fast beam losses was serious in the operation.

Solenoid coils were wound at the magnet free section to protect the electrons coming to the beam position. The field strength is \(<50\text{G}\). The solenoid coils are covered 95% of the magnet free section finally. The solenoid coil did not work well to suppress the coupled bunch instability, while it works to suppress the single bunch instability very well. Electrons stay longer time in the vacuum chamber than that for the case of no solenoid. R/Q of the wake force induced by electron cloud was reduced, but the range of the wake field (Q) was longer. Bunch-by-bunch feedback system suppressed the coupled bunch instability for wider spacing >6ns.

Systematic measurement and simulations in KEKB were published in [4,5], respectively. Motion of electrons is different between in drift space or in the weak solenoid field (\(<50\text{G}\) ). Electrons rotate along the chamber wall and do not approach to the beam. The electron motion reflects the instability signal.

Figure 7 shows the horizontal mode spectrum for solenoid OFF. The left picture was given by measurement. The right picture was given by simulation for electron cloud in drift space. Vertical spectra for measurement and simulation were similar as horizontal ones in Figure 7.

Figure 8 shows the horizontal mode spectrum for solenoid ON. The left and right pictures are given for measurement and simulation. Vertical spectra for measurement and simulation were similar as Figure 8.

The spectra with and without solenoid had very good agreement with simulations.
Figure 7: Horizontal mode spectrum in KEKB. Left picture is given by measurement with solenoid OFF [4,5]. Right picture is simulated by electron cloud in drift space.

Figure 8: Horizontal mode spectrum in KEKB. Left picture is given by measurement with solenoid ON [4,5]. Right picture is simulated by electron cloud in solenoid field 10G.

**Coupled Bunch Instability in DAFNE**

Strong horizontal coupled bunch instability has been observed in DAFNE [6]. The unstable mode was slowest one; \( f = (1 - \Delta \nu_h) f_0 \), where \( \Delta \nu_h \) is the fractional part of the horizontal tune. DAFNE is a small ring; its circumference is 98m. Electrons in bending magnet and damping wiggler seemed dominant for the instability. Simulation was performed for electrons moving in a strong bending field. Figure 9 shows growth of transverse amplitude and horizontal mode spectrum given by the simulation.

Electrons in bending magnets form stripes, because electrons are trapped along the vertical field line. The density of the stripe increases for increasing secondary emission rate. In the instability, the stripe and bunch train experience coherent motion, thus the slowest unstable mode \((m=114, f = (120-m-\nu_h) f_0)\) is induced as shown in Figure 9.

Figure 9: Growth of transverse amplitude and horizontal mode spectrum given by the simulation.

Figure 10 shows two cut of movie of beam-stripe motion. The white point is the position where bunch passes in the rectangular chamber; the wiggler chamber of DAFNE is the size 120mmx40mm. When a bunch passes though with a left deviation, the stripe deviates to right in left picture, vice versa in right picture. In the movie the stripe motion oscillates with a delay (less than \( \pi \)) for the beam motion.

**SINGLE BUNCH INSTABILITY DUE TO ELECTRON CLOUD**

Single bunch fast head-tail instability caused by electron cloud has been observed in KEKB. When beam current exceeds a threshold value, emittance increases and synchro-beta side band signal has been observed.

The simulation of the fast head-tail instability is performed by solving following equations [7]

\[
\frac{d^2x_p}{ds^2} + K(s)x_p = \frac{r_e}{\gamma} \frac{\partial \phi_p(x_p)}{\partial x_p} \delta_p(s - s_e)
\]

\[
\frac{d^2x_e}{dt^2} = \frac{e}{m_e} \frac{dx_e}{dt} \times B - r_e c^2 \frac{\partial \phi_p(x_e)}{\partial x_e} \delta_p(t - t_p(s_e))
\]

Each potential of beam and electrons are solved using PIC algorithm. Since beam (1mmx0.1mm) is localized at the chamber center, free boundary condition is employed. Electron cloud is initialized every interactions with beam with a flat distribution \( \sim 40 \sigma_x \times 60 \sigma_y \).

The single bunch instability signal has been also observed in PETRA-III and Cesr-TA, recently.

**Single Bunch Instability in KEKB-LER**

A beam size blow-up had been observed since early stage of KEKB operation around ~1999 [8]. The blow-up limited the luminosity performance. Figure 11 shows beam size blow-up and luminosity limitation in 2000-2001. Solenoid coils were wound 2000-2001 in 50%-70% of the whole drift space. Left picture shows beam size blowup without (green) and with (red) the weak solenoid field. Threshold of the beam size blow-up increases 400 to 800mA. Left picture shows luminosity as function of current. Luminosity was saturated around 550mA at 2000 December. After winding solenoid additionally, 50% to 70%, luminosity was not saturated by 700mA at 2001 March. The solenoid coil was wound further after 2001, and was covered 95% of drift space at around 2005.

Figure 11: Beam size blow-up and luminosity limitation in 2000-2001.
Synchro-betatron sideband signal, which indicates head-tail instability caused by electron cloud, has been observed [9]. The synchro-beta signal synchronizes with the beam size blow-up: when the beam size blow-up is suppressed by the solenoid, the sideband disappears, vice versa. Figure 12 shows the betatron and synchrotron sideband spectra along the bunch train. Vertical axis is bunch train. Betatron signal, which is left white line, shift positively. It is tune shift due to electron cloud. Right white line is a positive synchrotron sideband, whose frequency is \( v_r + \alpha v_s \), where \( 1 < \alpha < 2 \). The separation of betatron and sideband is larger than \( v_s = 0.025 \), because of the fast head-tail instability; perhaps mode coupling of \( m=1 \) and \( m=2 \).

**Figure 12**: Measured spectrum for vertical bunch motion. Lower and upper is FFT signal of head and tail bunches [9].

Figure 13 shows simulation result of the fast head-tail instability using Eq. (3). Beam size evolution and FFT of the bunch motion are depicted in top and bottom pictures. The figure shows the threshold electron density is around \( 8\times10^{11} \) m\(^{-3} \). Upper synchrotron sideband appears above \( 10^{12} \) m\(^{-3} \). The separation from betatron tune is larger than betatron tune 0.025. The result agrees well with the measurement.

**Figure 13**: Simulation result of the fast head-tail instability using Eq. (3). Top picture shows beam size growth for various electron densities. Bottom picture shows FFT of the bunch motion in the top picture.

**Single Bunch Instability in PETRA-III and Cesr-TA**

The single bunch instability has been observed in PETRA-III and Cesr-TA. PETRA-III is synchrotron light source with very low emittance (\( \epsilon_x = 1 \) nm). The electron cloud instability is observed at multi-bunch operation with a narrow bunch spacing 16 ns [11]. They have observed upper sideband signal, which separates from betatron signal \( \nu_r \). The measurement in PETRA-III agrees well with that in KEKB.

Cesr-TA is operated to study the electron cloud effect in linear collider positron damping ring with very low emittance. The sideband signal observed in Cesr-TA is negative sideband of betatron signal; \( \nu_r \nu_s \) [12]. The separation from the betatron tune is just the synchrotron tune \( \nu_s \). Simulations, which were performed for Cesr-TA, showed that negative sideband was dominant in the low emittance [14].

**SUMMARY AND CONCLUSIONS**

Mode spectra observed in the coupled bunch instability are clear evidence of the electron cloud effects. Corrective motion of electrons interacting with beam reflects to the unstable mode spectra. Electrons moving in drift space induces broad modes higher than \( m-h/2 \). Electrons in weak solenoid field induces slow and positive broad modes \( m-t_0 \). Electrons in bending field causes instabilities slow mode \( m=t_0 \).

Synchro-betatron sideband spectra in the single bunch instability appear as positive sideband in KEKB and PETRA-III, while negative sideband in Cesr-TA. Simulations can give consistent results as the measurement, but underlying physics is not clear.

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**REFERENCES**