Bayesian Reliability Model for Beam Permit System of RHIC at BNL

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DID THE SUN JUST EXPLODE?  
(ITS NIGHT, SO WE'RE NOT SURE)

This neutrino detector measures whether the sun has gone nova.

Then, it rolls two dice. If they both come up six, it lies to us.
Otherwise, it tells the truth.

LET'S TRY.

DETector! Has the sun gone nova?

(ROLL)

YES.

FREQUENTIST STATISTICIAN:
The probability of this result happening by chance is $P = 0.0027$.

Since $P < 0.05$, I conclude that the sun has exploded.

BAYESIAN STATISTICIAN:
Bet you $50 it hasn't.

COURTESY: HTTP://XKCD.COM/1132/
RHIC

- Relativistic Heavy Ion Collider – 2.4 mile
- Yellow and Blue – two counter circulating beams
- 6 interaction regions

Relativistic Heavy Ion Collider
Beam Permit System

- RHIC peak stored energy 72 MJ
  - 70 MJ in SC magnets
  - 2 MJ in beams
- Beam Permit System (BPS):
  - Receives subsystems’ health inputs
  - Takes decision for disposal of energy
- Impacts reliability and availability of RHIC
Objective

- Estimate Reliability characteristics of BPS - Draw best possible inference
- Reliability Information Sources
  1. Monte Carlo model\(^1\): Profound view
     - Basic component failures\(^2\), Structural bottlenecks
     - Exponential – MIL HDBK\(^3\), Manufacturer’s data
  2. Historical failure data (15 years): Bird’s eye view
     - Real survival characteristics

HOW TO CONSIDER BOTH?
Integration of information from multiple sources

Updating current knowledge when new information is acquired

\[ f(x \mid \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Frequentist Approach
- Event probability: limit of its long term occurrence rate
- Parameters constant - \( \mu, \sigma \)

Bayesian Approach
- Keeps updating probability
- Parameters are random variables - \( \mu, \sigma \)

Bayesian Paradigm\textsuperscript{[4]}


Bayesian reliability model for beam permit system of RHIC at BNL

Thomas Bayes
Bayesian Paradigm

Bayes theorem: Continuous form

\[
\pi(\theta|x) = \frac{L(\theta|x) \times \pi(\theta)}{f(x)}
\]

Unknown parameter of a distribution
New recorded data
Likelihood function
Prior distribution
Posterior distribution
Marginal distribution of \(x\)

\(\theta \sim N(\nu, \omega^2)\)

\[\pi(\theta|x) \propto L(\theta|x) \times \pi(\theta)\]

Foundation of Bayesian analysis


Bayesian reliability model for beam permit system of RHIC at BNL
Source 1: Monte Carlo model

- Exponential \[^5\] survival distribution of components
- Simulates system behavior – top distribution may be different

**STEP 1**

Check - Non homogenous Poisson process\[^6\]

\(\lambda\) for \(10^7\) hours – no trending

**STEP 2**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameter</th>
<th>Point estimate</th>
<th>AIC*</th>
<th>BIC**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>Scale - (\lambda)</td>
<td>8.831e-5</td>
<td>3141125.52</td>
<td>3141136.15</td>
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<tr>
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<td>Shape - (\alpha)</td>
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*Akaike Information Criterion\[^7\]
**Bayesian Information Criterion\[^7\]
Source 2: Historical failure data

- BPS hardware failure data for 15 years
- 16 data points for the time between failures owing to high reliability of BPS

Check - Distribution of historical data with goodness-of-fit

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<th>BIC**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
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<td>Lognormal</td>
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<td>Shape - ( \alpha )</td>
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</tbody>
</table>

*Akaike Information Criterion**Bayesian Information Criterion
Bayesian reliability model

- **Posterior**
  - Tradeoff between Prior and Data distribution
  - Tradeoff level: depends on relative strength, can be changed by altering hyperparameters

- Conjugate prior: Yields a posterior of the same form as the data distribution

- Model choice:
  - Data - Weibull
  - Prior – Exponential (Weibull with shape 1)
  - Bayesian – Weibull (Scale and shape unknown)
Data Distribution \[8\]

Weibull: \( f(x|\alpha, \eta) = \alpha \eta x^{\alpha-1} e^{-\eta x^\alpha} \)

- Shape = \( \alpha \)
- Scale = \( \eta^{-1/\alpha} \)

Likelihood function:
\[
L(\alpha, \eta|x) = \prod_i^k \alpha \eta x_i^{\alpha-1} e^{-\eta x_i^\alpha}
\]

\( \alpha = 0.6275, \eta = 1.2904 \)
\( \alpha, \eta \sim [0, \infty) \)
Conjugate Prior Distribution \[ [8] \]

Joint prior distribution for $\alpha, \eta$

$$p(\alpha, \eta) \propto e^{-\alpha} \eta^\beta - 1 e^{-\eta}$$

Hyperparameter $\beta = 3$

$$\alpha = 1, \eta = 0.7741 \quad \alpha, \eta \sim [0, \infty)$$
Hyperparameter choice

Hyperparameter $\beta = 3$
$\eta = 0.7741, \alpha = 1.000$
Posterior parameter samples [4,8]

- Posterior kernel: Contains all the information of $\alpha$ and $\eta$
  \[ p(\alpha, \eta|\mathbf{x}) \propto \alpha^k \eta^{k+\beta-1} \left( \prod_i^k x_i^\alpha \right)^{\alpha-1} e^{-\eta \sum x_i^\alpha - \alpha - \eta \beta} \]

- Proposal density
  \[ q(\alpha', \eta'|\alpha, \eta) = \frac{1}{\alpha \eta} e^{\left( \frac{\alpha'}{\alpha} - \frac{\eta'}{\eta} \right)} \]

- Metropolis Hastings Algorithm
  \[ a((\alpha', \eta'), (\alpha, \eta)) = \min \left\{ 1, \frac{p(\alpha', \eta')}{p(\alpha, \eta)} \frac{q(\alpha, \eta'|\alpha, \eta)}{q(\alpha', \eta'|\alpha, \eta)} \right\} \]

- Posterior parameters
  - $\alpha = 0.6327$
  - $\eta = 1.2225$
Posterior Distribution

Prior: $\alpha = 1.0000, \eta = 0.7741$
Data: $\alpha = 0.6275, \eta = 1.2904$

Posterior: $\alpha = 0.6327, \eta = 1.2225$
Discussion

Hyperparameter $\beta = 3$

$\alpha = 0.6327$
$\eta = 1.2225$

Hyperparameter $\beta = 15$

$\alpha = 0.6404$
$\eta = 1.1249$

Prior: $\alpha = 1.0000, \eta = 0.7741$
Data: $\alpha = 0.6275, \eta = 1.2904$

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Conclusion

- Bayesian analysis furnishes the most informed inference for BPS

- Emphasize the importance of both the information sources
  - MC Model - Fine failure characteristics
  - Historical data - Real survival behavior

- Advocate the value of $\beta = 3$
  - High confidence in the actual machine failure data
  - Mild influence of the MC model results

- Ability to regulate the influence of either information sources
Why?

Bayesian reliability model for beam permit system of RHIC at BNL
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  Controls Hardware Group, Collider-Accelerator Department, Brookhaven National Laboratory, NY
References

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2. P. Chitnis et al., “Quantitative fault tree analysis of the beam permit system elements of RHIC at BNL”, ICALEPCS 2013, San Francisco
4. S. T. Rachev et al., Bayesian Methods in Finance, 2008, Wiley
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“Is this needed for a Bayesian analysis?”

Image courtesy: http://capewest.ca/cartoons.html

Thank You!

Questions?