UNCERTAINTY MODELLING OF RESPONSE MATRIX
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UNCERTAINTY DESCRIPTIONS

- The difference between the ideal and real response matrices of a storage ring is referred to as uncertainty.
  \[ \Delta_R = R_{\text{ideal}} - R_{\text{real}} \]
- The uncertainty in response matrix combines BPM scaling errors, corrector scaling errors and tune drift and it is not straightforward to distinguish one source of error from the other.
- Instead it is useful to model uncertainties in process outputs (BPM values) and process inputs (Corrector values).
- Using a Singular Value Decomposition (SVD): \[ R = \Phi \Sigma \hat{\Psi}^T \]
- Using Fourier analysis: \[ R = \Phi \Sigma \hat{\Psi}^T \]

UNCERTAINTY DESCRIPTIONS USING SINGULAR VALUE DECOMPOSITION

- Uncertainty in BPMs is expressed by comparing the \( \Phi \) matrices of the ideal and real response matrices.
- The shape of the uncertainty has a strong diagonal component but there are also non-diagonal effects meaning that there is correlation to other BPMs in these modes.
- Uncertainty in the correctors is expressed by comparing the \( \Psi \) matrices of the ideal and real response matrices.
- The shape of the uncertainty has a strong diagonal component but there are also non-diagonal effects meaning that there is correlation to other correctors in these modes.
- Uncertainty in the singular values can be expressed by comparing the \( \Sigma \) matrices of the ideal and real response matrices which is diagonal.
- The size of the uncertainty in the higher order modes is greater than the error in low order modes meaning that the high order modes are more vulnerable to stability problems.

UNCERTAINTY DESCRIPTIONS USING HARMONIC DECOMPOSITION

- Uncertainty in the process outputs can be expressed by comparing the \( \Phi \) matrices of the ideal and real response matrices.
- The matrix \( \Phi \) depends solely on the beta function and phase advance at BPM locations. Coupling between the beta function and phase advance within a cell results in off-diagonal elements being non-zero.
- Uncertainty in the process inputs can be expressed by comparing the \( \Psi \) matrices of the ideal and real response matrices.
- The matrix \( \Psi \) depends solely on the beta function and phase advance at corrector locations. Coupling between the beta function and phase advance between cells results in off-diagonal elements being non-zero.
- The uncertainty in the Fourier coefficients is diagonally structured and the largest error is seen at the modes associated with the tune i.e. the 13th mode for tune = 13.36. The harmonic modes closest to the tune are more vulnerable to stability problems.
- The Fourier coefficients are solely determined by the tune, therefore the size of uncertainty in the tune is determined for each harmonic mode.

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