DIRECT OBSERVATION OF ULTRALOW VERTICAL EMITTANCE USING A VERTICAL UNDULATOR

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Abstract

In recent work, the first quantitative measurements of electron beam vertical emittance using a vertical undulator were presented, with particular emphasis given to ultralow vertical emittances [K. P. Wootton, et al., Phys. Rev. ST Accel. Beams, 17, 112802 (2014)]. Using this apparatus, a geometric vertical emittance of $0.9 \pm 0.3$ pm rad has been observed. A critical analysis is given of measurement approaches that were attempted, with particular emphasis on systematic and statistical uncertainties. The method used is explained, compared to other techniques and the applicability of these results to other scenarios discussed.

INTRODUCTION

The low emittance ring community expects direct observation of beam size as demonstration of ultralow vertical emittance in electron storage rings. In particular, the development of low emittance tuning routines at electron storage rings for proposed linear collider damping rings has motivated the measurement of pm rad vertical emittances. A variety of techniques for measuring vertical emittance have been developed, typically utilising imaging, interferometry or projection of the distribution of spontaneous synchrotron radiation produced by the electron beam.

Recent experiments and simulations have demonstrated that undulator radiation from a vertical insertion device is particularly sensitive to pm rad vertical emittance [1]. However the use of a vertical undulator beamline for direct measurement of pm rad vertical emittance in a storage ring presented several challenges. This work is a critique of several experimental approaches to the measurement of vertical emittance using a vertical undulator.

THEORY

The use of a vertical undulator for measurement of vertical emittance in an electron storage ring was first proposed by S. Takano in 1997 [2]. Using simulations, it was demonstrated that a measurement of the on-axis flux from a short vertical insertion device could be used to evaluate the vertical emittance in the SPring-8 storage ring.

The spectral brilliance of a planar undulator yields a distribution with odd harmonics of high intensity, and null even harmonics. The angular distribution of radiation for the first harmonic illustrated in Fig. 1(a) of Ref. [3] could be approximated by a Gaussian distribution. However, using the first harmonic for a vertical emittance monitor with an opening angle of order $\approx 1/\gamma$ limits the minimum electron beam emittance which can be deconvolved from a measured photon distribution to approximately the same order.

This approximation breaks down at high undulator harmonics. High harmonics yield an angular distribution of undulator radiation which can be described as the fine structure of a narrow interference pattern within the usual cone of undulator radiation. This pattern exhibits minima on axis for even harmonics, and maxima on axis for odd harmonics, and enables measurement of emittances smaller than the undulator radiation opening angle. Employing a narrow interference pattern convolved with the electron beam distribution, this technique is similar to several other emittance diagnostics, such as the $\pi$-polarisation technique [4], synchrotron radiation interferometer [5], X-ray Fresnel diffraction [6] and the coded aperture X-ray emittance monitor [7].

PREVIOUS MEASUREMENTS OF ULTRALOW VERTICAL EMITTANCE

Vertical emittance measurements in 2008 at the SLS using the $\pi$-polarisation technique demonstrated $\epsilon_y = 3.2 \pm 0.7$ pm rad [4].

Experiments conducted in 2010 using the AS storage ring demonstrated through indirect measurements a vertical emittance of $\epsilon_y = 1.2^{+0.3}_{-0.2}$ pm rad [8].

In 2012, a new vertical emittance of $\epsilon_y = 0.9 \pm 0.4$ pm rad was observed using the direct $\pi$-polarisation technique at the SLS storage ring [9].

With the goal of optimising the AS storage ring for lower vertical emittance, a beam-based survey of storage ring magnets was undertaken [10], culminating in 2012 in the mechanical alignment of individual sextupole magnets within vertical tolerances of $\Delta y < \pm 25 \mu$m [11]. Indirect measurements of the bunch volume by the Touschek lifetime demonstrated vertical emittances below 1 pm rad [11].

The goal of the work presented in [3, 12] was direct measurement of picometre electron beam vertical emittance beams at the AS storage ring.

MEASUREMENT APPROACHES

The flux ratio of the 14th to 15th harmonics was measured using the approaches of energy scans, time-averaging and electron beam orbit bumps. These harmonics were selected as they were the highest undulator harmonics (greatest sensitivity to vertical emittance) which were still lower in photon energy than the Au absorption edge cutoff of 2150 eV for the beamline, which arises from Au coatings on the beamline mirrors [13].

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Photon Energy Spectrum

Initial experiments were conducted by scanning the beamline monochromator [1, 14]. The positions of four blades were optimised to minimise the size and vertical position of the on-axis pinhole passing undulator radiation. In those experiments, the size of the pinhole was not measured, hence the emittance could not be directly inferred. Instead, the vertical emittance of the electron beam was set based on model values determined using LOCO [15].

In practice, it was not possible to simultaneously minimise the position and size of the pinhole within the tolerance required to directly measure emittances. The fitted pinhole dimension and position were the principal systematic uncertainty limiting the direct measurement of vertical emittance.

Time Averaging

One way to minimise the statistical uncertainty in a given measurement is to increase the number of acquisitions of that quantity. For a given mean of the sample population μ, the statistical uncertainty δμ for n acquisitions is [16]

δμ = \frac{σ}{\sqrt{n}},

where σ is the standard deviation of the sample population. This approach was used effectively to minimise the statistical uncertainty in the measurement of vertical emittance [17]. However, the systematic uncertainty arising from the pinhole position remained.

Blade Scans

Instead of optimising for a pinhole dimension, a single blade was scanned vertically through the undulator radiation distribution [18]. The vertical angular distribution of undulator radiation was recovered from by numerically differentiating the transmitted intensity with respect to vertical position. The recovered and simulated vertical distribution of undulator radiation for vertical emittances of 100 pm rad and 1 pm rad are plotted in Fig. 1.

Figure 1 shows that there is a small difference in the measured radiation distributions for significantly different vertical emittances. However, this technique is not sufficiently sensitive to discriminate emittances on the order of pm rad.

Orbit Bumps

Instead of optimising for both the centred position and minimum pinhole size, the decision was made to optimise only the vertical pinhole dimension. The blade positions were optimised with the beamline monochromator aligned to pass a high, odd undulator harmonic (the 15th). With one blade positioned near the centre of the undulator radiation distribution, the second was closed by moving vertically in steps. As illustrated in Fig. 3 of Ref. [3], it was possible to close the two blades to a sensible pinhole height of 5 ± 5 μm. However, this minimised pinhole size was not necessarily correctly positioned to pass the centre of the undulator radiation distribution.

To scan through the centre of the undulator radiation distribution, vertical orbit bumps were made across the two sectors of the AS storage ring adjacent to the insertion device. Figure. 10 of Ref. [3] illustrates the trajectory arising from a four-corrector closed orbit bump which results in a vertical angle kick through the insertion device. The vertical distribution of undulator radiation was measured passing the optimised pinhole by scanning the bump magnitude of the electron beam in the storage ring.

RESULTS

The flux ratio of the 14th to 15th harmonics was measured using the several approaches outlined. The AS storage ring was set using LOCO with lattices of various ultralow vertical emittances below 100 pm rad, and the vertical emittance was measured at these setpoints. Measurements ultralow vertical emittance using orbit bumps were presented in Ref. [3]. Here, measurements using energy scans and time averaging are also compared, and presented in Figs. 2 and 3.
The flux ratio for various emittance setpoints was simulated using SPECTRA [19], importing the measured magnetic field map of the insertion device [20].

The most successful measurement technique was orbit bumps through the insertion device. As outlined in Ref. [3], the smallest vertical emittance measured using the vertical undulator technique was \( \varepsilon_y = 0.9 \pm 0.4 \) pm rad. The vertical emittance measured using LOCO, with the quantum limit of vertical emittance [21] added in quadrature was \( \varepsilon_y = 0.4 \pm 0.1 \) pm rad. Although these direct and indirect measurements disagree within uncertainty, the vertical emittance measured using the vertical undulator is equal to the world-record low of Ref. [9].

DISCUSSION

Diffraction-Limited Storage Rings

Brilliance-optimised diffraction-limited storage rings are currently under construction or proposal in several countries. A survey of horizontal and vertical emittances of current and proposed electron and positron storage rings is presented in Fig. 4.

![Figure 3: Measured flux ratio \( F_{14}/F_{15} \) compared for several measurement approaches, for vertical emittances below 15 pm rad.](image)

![Figure 4: Survey of horizontal and vertical emittances of several current (measured) and proposed electron and positron storage rings. The measurement of Ref. [3] is denoted by c.](image)

The designs call for orders of magnitude increase in storage ring brightness compared to existing third-generation storage ring facilities. Principally, this will be achieved with insertion device beamslines in multi-bend achromat storage ring lattices with horizontal emittances below 500 pm rad. This emittance regime has been tested using vertical insertion devices in existing storage rings.

Of particular interest are measurements of the angular distribution of undulator radiation at high undulator harmonics. For ultralow vertical emittances, the angular profile of undulator radiation departs significantly from typical Gaussian distributions, resulting in a narrow diffraction pattern, as highlighted in Fig. 3 of Ref. [18]. Photon beamlines utilising high-undulator harmonics at diffraction-limited storage rings should carefully evaluate this departure from Gaussian approximated spatial distribution of photon beams at existing storage ring facilities.

Emittance Growth Resulting from Undulator Self-Dispersion

It is well-known that the vertical dispersion of a lattice increases the equilibrium vertical emittance. Vertical emittance optimisation routines seek to minimise coupling and dispersion terms simultaneously. The inclusion of a vertical insertion device in a lattice has the well-known effect of increasing vertical emittance, which lattice designs have exploited with the intent of creating round beams [22]. This suggests that designs for ultralow vertical emittance avoid introducing vertical dispersion.

Vertical emittance growth due to undulator self-dispersion was calculated according to the method of Ref. [22], for the APPLE-II undulator and the normal AS user lattice with 0.1 m distributed horizontal dispersion in the insertion straights. The vertical emittance growth in a ring from self-dispersion in a vertical wiggler is given by [22],

\[
\Delta \varepsilon_y = \frac{5 \pi \beta_y(s)}{6} \frac{\rho_0}{\rho_w} \left( \frac{\rho_0}{\rho_w} \right)^2 \frac{N_w \theta_w}{1 + \frac{2}{\pi} \frac{\rho_0}{\rho_w}}.
\]

For a ring, the mean of the curly-\( \mathcal{H} \) function can be approximated by [22],

\[
\langle \mathcal{H}_0 \rangle = \frac{\varepsilon_x \rho_0 \rho_0}{C_a E^2 R} \int F_x. \tag{3}
\]

Using the parameters in Table 1 for this experiment at the AS, the curly-\( \mathcal{H} \) function was evaluated as \( \langle \mathcal{H}_0 \rangle = 0.0027 \) m rad.

For the parameters of the storage ring and undulator at the AS given in Table 1, the vertical emittance increase due to self-dispersion of Eq. 2 is presented in Fig. 5 for an increasing number of vertical undulator poles.

The APPLE-II undulator used at the AS has a total of 50 poles, giving a calculated increase in vertical emittance due to self-dispersion of \( \Delta \varepsilon_y = 0.012 \) pm rad. Increasing the vertical emittance to \( \varepsilon_y \approx 1 \) pm rad by self-dispersion
requires approximately 6500 undulator poles, at a total undulator length of 240 m. Hence it can be concluded that the increase in vertical emittance due to self-dispersion is negligibly small for practical devices in present storage ring light sources.

Table 1: Parameters Used in the Calculation of Vertical Undulator Self-Dispersion.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy $E_0$</td>
<td>3.0</td>
<td>GeV</td>
</tr>
<tr>
<td>Energy spread $\sigma_E$</td>
<td>0.11</td>
<td>%</td>
</tr>
<tr>
<td>Horizontal emittance $\varepsilon_x$</td>
<td>10</td>
<td>nm rad</td>
</tr>
<tr>
<td>Undulator period length $\lambda_u$</td>
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<td>mm</td>
</tr>
<tr>
<td>Peak field $B_u$</td>
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<td>T</td>
</tr>
<tr>
<td>Deflection parameter $K_u$</td>
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<td></td>
</tr>
<tr>
<td>Number of full periods $N_u$</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Number of poles $N_w$</td>
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<td></td>
</tr>
<tr>
<td>Deflection angle $\theta_w$</td>
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<td>rad</td>
</tr>
<tr>
<td>Bending radius $\rho_w$</td>
<td>29.0</td>
<td>m</td>
</tr>
<tr>
<td>Storage ring diameter $D$</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Damping decrement $\gamma$</td>
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<td></td>
</tr>
<tr>
<td>Ring radius $R$</td>
<td>34.4</td>
<td>m</td>
</tr>
<tr>
<td>Curly-$H$ function $\langle H_0 \rangle$</td>
<td>$2.3 \times 10^{-3}$</td>
<td>m rad</td>
</tr>
</tbody>
</table>

Figure 5: Increase in vertical emittance due to vertical undulator self-dispersion for an undulator of $N_w$ poles. The APPLE-II undulator at the AS has 50 poles.

**Emittance Growth Resulting From Orbit Steering**

Making use of the electron beam orbit correctors in the storage ring, it was possible to steer the electron beam with vertical angle bumps through the insertion device, which is counter-intuitive for a vertical emittance measurement. A four-corrector vertical angle bump was made across the two sectors of the storage ring adjacent to the APPLE-II insertion device, which steered the electron beam vertically off-axis through lattice quadrupoles and sextupoles. This has the effect of introducing skew betatron coupling and vertical dispersion, which are well-known to contribute significantly to vertical emittance.

The magnitude of local orbit bump required for the orbit bump measurements is of the order $< 10 \mu$rad. In Ref. [3], it was demonstrated that for a well-corrected storage ring lattice this would result in a negligible vertical emittance growth of $\varepsilon_y = 0.07 \pm 0.01$ pm rad.

**Ideal Vertical Insertion Device**

At the time of construction, the APPLE-II insertion device used was shimmed to correct multipole field errors while operating in the horizontal polarisation mode: operation as a usual horizontal undulator [20]. For operation principally as a vertical undulator, it would be beneficial to shim the insertion device for use in the vertical polarisation mode.

**Ideal Detector**

The principal uncertainty in the vertical undulator technique is the uncertainty in the pinhole vertical dimension and position. For future experiments to measure vertical emittance, there are two complementary directions for an ideal detector – either a single pinhole of fixed dimensions, or a pixel detector for profile measurements of the undulator photon beam. Future experiments to measure vertical emittance should consider using a pinhole of known diameter, as in other work to characterise a tandem APPLE-II undulator [23].

A pixel detector observing the angular distribution of undulator radiation at a fixed photon energy could be used to measure the vertical emittance in much the same way as orbit bumps through the insertion device. A candidate detector particularly appropriate to this photon energy range is the DiagOn device developed at SOLEIL. Recently, direct projections of undulator harmonics have been measured at SOLEIL [24]. Designed as a beam diagnostic for APPLE-II insertion devices operated in the horizontal polarisation orientation, the reported device measures the distribution of horizontally-polarised undulator radiation at a fixed photon energy. As the desired polarisation for vertical emittance measurement using a vertical undulators corresponds to vertical linear polarised radiation, the device would need to be rotated about the beam axis to pass photons of vertical polarisation.

**CONCLUSION**

Measurement of vertical emittance using a vertical undulator in an electron storage ring has been achieved. This is a direct measurement of emittance, based on the convolution of the angular divergence of the electron beam with the single-electron undulator radiation distribution. The smallest measured vertical emittance at the AS storage ring was $\varepsilon_y = 0.9 \pm 0.3$ pm rad.

These measurements of undulator radiation distributions with ultralow electron beam emittances have highlighted that the angular distribution of undulator radiation departs from usual Gaussian approximations, at high undulator harmonics.
This is a consideration that photon beamlines at proposed diffraction-limited storage ring light sources should be aware of.

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REFERENCES