LONGITUDINAL BUNCH PROFILE RECONSTRUCTION USING BROADBAND COHERENT RADIATION AT FLASH

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Abstract

The required high peak current in free-electron lasers is realized by longitudinal compression of the electron bunches to sub-picosecond length. Measurement of the absolute spectral intensity of coherent radiation emitted by an electron bunch allows monitoring and reconstruction of the longitudinal bunch profile. To measure coherent radiation in the terahertz and infrared range, a high-vacuum coherent radiation intensity spectrometer was developed for the free-electron laser in Hamburg (FLASH). The spectrometer is equipped with five consecutive dispersion gratings and 120 parallel readout channels: It can be operated either in short (5 - 44 µm) or in long wavelength mode (45 - 430 µm). Fast parallel readout permits the monitoring of coherent radiation from single electron bunches. Large wavelength coverage and superb absolute calibration of the device allows reconstruction of the longitudinal bunch profile from measured coherent radiation spectrums using a Kramers-Kronig based phase retrieval technique. The device is used as a bunch length monitor and tuning tool during routine operation at FLASH. Comparative measurements with radio-frequency transverse deflecting structure show excellent agreement of both methods.

INTRODUCTION

In order to measure the length of the electron bunches with femto second duration in a high-gain free-electron laser, a coherent intensity spectrometer with four stages (CRISP4) was developed at FLASH [1].

The emission process of an extended charge distribution can be described by a linear superposition of the electric fields of the N individual particles. The wavelength dependent energy density spectrum into a solid angle interval is given by

\[
\frac{d^2U}{d\omega d\Omega} = \left( \frac{d^2U}{d\omega d\Omega} \right)_1 \frac{N + N(N - 1) |F(\omega, \Omega)|^2}{1}
\]

with \(F(\omega, \Omega)\) the (complex) bunch form factor and \(\left( \frac{d^2U}{d\omega d\Omega} \right)_1\) the spectrum of a single charge. Coherent radiation dominates for \(|F(\omega, \Omega)|^2 \gg 1/N\). \(F(\omega, \Omega)\) is given by the Fourier transform of the normalized three dimensional particle density distribution \(S_{3D}\) as

\[
F(\omega, \Omega) = \int S_{3D}(\vec{r}) e^{-i\vec{k} \cdot \vec{r}} d\vec{r},
\]

where \(\vec{k}\) is the wave vector pointing in the direction of the observer (\(\vec{k} = \omega/c\)) and \(\vec{r}\) the position vectors of the individual charges. In case of uncorrelated longitudinal and transverse charge distributions, the form factor can be factorized as

\[
F(\omega, \Omega) = F_L(\omega, \Omega) F_T(\omega, \Omega).
\]

For highly relativistic bunches (\(\gamma \gg 1\)), the coherent radiation is confined in a narrow cone in forward direction. Restriction of the observation direction to small angles strongly suppresses the influence of the transverse bunch structure on the form factor. In this case, the frequency dependence of the form factor is predominantly determined by the longitudinal structure of the bunch

\[
F(\omega, \Omega) \propto F_L(\omega) = \int S(t) e^{-i\omega t} dt.
\]

A measurement of the coherent radiation spectrum yields the absolute magnitude of the form factor as a function of wavelength but the phase remains unknown. Hence the determination of the longitudinal charge distribution by inverse Fourier transformation is not directly possible. Phase information can be obtained with the help of the Kramers-Kronig relation [2] if a sufficient wavelength range is covered.

TEMPORAL PROFILE RECONSTRUCTION

As shown in Eq. (1) to (4), the longitudinal charge distribution is related to the spectral density of the coherent radiation through the longitudinal form factor \(F_L(\omega)\). For very low frequencies, corresponding to wavelengths much longer than the bunch length, all electrons radiate coherently. The form factor is a real number and approaches unity for \(\omega \to 0\). For wavelengths comparable and shorter than the bunch length, the longitudinal form factor is a complex number

\[
F_L(\omega) = |F_L(\omega)| e^{i\phi(\omega)}
\]

with \(|F_L(\omega)| < 1\). If both, \(|F_L(\omega)|\) and \(\phi(\omega)\) were known, the longitudinal charge profile \(S(t)\) could be reconstructed by inverse Fourier transform. From spectroscopic measurements, only \(|F_L(\omega)|\) can be derived while the phase remains unknown.

Generally it is not possible to reconstruct the phase from the modulus of a 1-dim complex function in an unique way even if this function is causal [3]. Nevertheless, it is possible to compute the minimal phase using the Kramers-Kronig relation which connects the real and imaginary parts of the Fourier transform of a causal function.

\[
S(t) = \frac{1}{2\pi} \int F_L(\omega) e^{i\omega t} d\omega.
\]
method has been adopted for the phase reconstruction of
the complex bunch form factor by Lai and Sievers [4]
where it has been shown that $F_L(\omega)$ can be analytically
continued into the upper half complex plane, and the minimal
phase can be computed from the modulus of $F_L$ by means of the following integral

$$
\phi_{\text{min}}(\omega_0) = -\frac{2\omega_0}{\pi} \int_0^\infty \frac{\ln(|F_L(\omega)|) - \ln(|F_L(\omega_0)|)}{\omega^2 - \omega_0^2} d\omega.
$$

(6)

If the modulus of the form factor is experimentally
known over a sufficiently large wavelength range, covering
the main features of the bunch structure, Eq. (6) allows
to determine the minimal phase and thus reconstruct a
possible complex form factor using Eq. (5). Then the lon-
gitudinal charge distribution is calculated by a subsequen
tFourier transformation.

The reconstructed bunch shape is not unique and has to
be treated as one possible bunch shape. The reconstructed
bunch shape is the shape with the least complex substructure
compatible with the measured coherent radiation spec-
trum. In spite of these shortcomings, the phase reconstruc-
tion method yields very valuable information and allows to
a wide extend a reliable reconstruction of the bunch shape
as will be demonstrated in the next sections.

**EXPERIMENTAL SETUP**

The CRISP4 is part of the longitudinal diagnostics at
FLASH (Figure 1). Two spectrometers, located just be-
hind the last accelerator structure and in front of the un-
dulator magnets, are available for measurements. At both
locations an arbitrary bunch can be kicked out of the bunch
train with a fast kicker magnet to an off axis transision ra-
diation screen. The produced coherent transition radiation
is then transported to the spectrometer through evacuated
beam pipes. For comparison measurements in time do-
main, a transverse deflecting structure (TDS) located just
in front of the undulator magnets is used [5, 6].

To disperse polychromatic coherent radiation into its
spectral components, reflective gratings are used in our
setup. The spectrometers are equipped with five consec-
utive gratings, G0 to G4 (Figure 3), in a specific geome-
try [7]. The spectral intensity is recorded simultaneously
in 120 wavelength bins. Two remotely adjustable grating
sets are available to cover the mid-infrared (5-44 µm) and
the far-infrared (45-450 µm) range.

Determination of the absolute value of the form factor
requires a very detailed understanding of the whole mea-
urement setup starting from the characteristic of the radia-
tion process itself to the absolute spectral sensitivity of the
detectors. Emission characteristic of the source and trans-
port of the radiation to the detectors are calculated using

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**Figure 1:** Schematic overview of FLASH: Radio-frequency (RF) gun, superconducting acceleration modules (ACC), third-harmonic module (M3), two magnetic bunch compressors (BC), transverse deflecting structure (TDS) and coherent radiation intensity spectrometer with four stages (CRISP4).

**Figure 2:** Spectrometer response function for both grating sets at ADC level for $F_L = 1$, a transverse Gaussian profile with 200 µm (RMS) and a bunch charge $Q$ of 1 nC. Taken from [1].

**Figure 3:** Spectrometer layout with five consecutive grat-
ings: The gratings G1 - G4 consecutively disperse shorter
wavelengths in backward direction onto focusing 90°
ring mirrors and reflect longer ones to the next grating. A
filter grating G0 suppresses higher order effects.
Figure 4: Form factor calculated from the temporal profile measured with the TDS and measured with the CRISP4 for the same compression setting. Corresponding temporal profiles are shown in Figure 5(a).

dedicated simulation code named THzTransport [8]. The absolute calibration of the pyroelectric detectors for the low wavelength range 5-110 µm was done at the Free Electron Lasers for Infrared eXperiments (FELIX) [9]. For calibration of the spectrometer in the long wavelength range 110-450 µm, a theoretical model and complementary measurements with the TDS were used. The final wavelength depended response \( R(\lambda) \) of the spectrometer for an infinite short bunch and a charge of 1nC is shown in Figure 2.

The longitudinal form factor modulus is computed according to

\[
|F_L(\lambda)| = \sqrt{\frac{S_{SP4}(\lambda)}{Q^2 R(\lambda)}} \tag{7}
\]

with \( Q \) the charge of the bunch and \( S_{SP4} \) the ADC signal from the detectors.

Further technical information can be found in [1].

**MEASUREMENTS**

Due to the fact that for the transition radiation spectrometers individual electron bunches are kicked to off-axis TR screens, it is not possible to measure the spectra with both spectrometers from the same bunch. This is true as well for the direct TDS time domain measurement. It has been verified that the fluctuations of the bunch shape for adjacent bunches from the same bunch train are small, making a direct comparison of the bunch profile measured with all three devices meaningful.

A first comparison can be done on the level of the form factors to avoid the problem of phase reconstruction. Figure 4 shows the directly measured form factors from both spectrometers in comparison with the form factor derived from the TDS profile by Fourier transformation. All three form factors agree very well both in absolute magnitude and wavelength dependence. Down to wave-

Figure 5: Temporal profiles measured with the TDS and reconstructed with help of the Kramers-Kronig based phase retrieval technique from CRISP4 measurements for three different compression settings. Form factors of the profiles in Figure 5(a) are shown in Figure 4.
lengths of about 20 µm, even the fine structure is reproduced by the complementary methods. It should be mentioned that none of the methods is truly single shot, despite the fact that data (spectra in case of CRISP4, camera images in case of the TDS) for single bunches can be measured. The spectrometers require two measurements with different grating sets to cover the full range of wavelength (2 orders of magnitude) while the TDS method always requires at least measurements for two opposite streak directions to overcome the possible problem of longitudinal-transverse correlations inside the bunch.

Figure 5 compares bunch profiles in the time domain for three different compression settings. The TDS profiles are reconstructed by two-point-tomography using opposite streak directions as mentioned above. The profiles reconstructed from the spectrometer data use the Kramers-Kronig phase (minimal phase) as described in the section above. Due to the coverage of two decades in wavelength, basically no extrapolation of the measured form factors is needed. At long wavelengths, the measured form factor is very close to one (the limit for infinite wavelength) while at short wavelengths it is so small that the contribution to the reconstructed phase is negligible.

A comparison of the reconstructed bunch shapes from both spectrometers shows excellent agreement, despite the fact that the two devices are located about 60 m apart along the accelerator. While CRISP4 is located outside the tunnel and connected to the coherent transition radiation (CTR) screen by a 20 m long beamline containing seven optical elements (off-axis mirrors) [8], CRISP4-2 is located downstream the accelerator inside the tunnel. Furthermore, different types of pyroelectric detectors are used in both instruments. The excellent agreement of the reconstructed bunch lengths and structures demonstrates the excellent calibration and understanding of the spectrometers. This is further emphasized by the independent TDS measurement, both in absolute value of the peak current and the length and structure of the bunch.

Looking into more detail, there are still systematic differences visible which clearly need further study. Negative currents for instance in the phase reconstructed profiles are due to the “minimal phase” problem and can potentially be avoided by applying additional boundary conditions to the phase retrieval algorithm.

SUMMARY

The technology of bunch profile determination using coherent radiation spectroscopy has made a considerable step forward by using the multi-stage spectrometers developed at DESY. For the first time, we used two independent devices at different locations of the linac to cross check the reliability of the method and bench marked the result with direct measurements in time domain. The spectrometers are rather easy to use versatile instruments to permanently monitor and adjust the bunch profile at FLASH. Similar spectrometers will be used at FLASH-2 and the European XFEL.