Thresholds of the Head-Tail Instability in Bunches with Space Charge

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SIS100:
protons 4-29 GeV \( N_p = 2 \times 10^{13} \)
\( ^{238}\text{U}^{28+} \) 0.2-1.5 GeV/u \( N_p = 5 \times 10^{11} \)

One of the main concerns:
\( \text{U}^{28+} \) bunches can be head-tail unstable.
(High-Intensity long-time 1s accumulation)

ISIS Synchrotron of RAL, UK:
unstable head-tail modes produce losses in operation.
One of the concerns, especially for the Upgrade.

Theoretically, head-tail modes have no thresholds.

In reality, things are different and more complicated:
Transverse Space-Charge (nonlinear own-field), Machine Nonlinearities, Beam pipe, different 2RF buckets, nonlinearities in 1RF buckets.
Spallation Neutron Source
RAL, near Oxford, UK

C=163m
50 Hz RCS operation
up to $3 \times 10^{13}$ ppp
$Q_h = 4.31 - 4.18$
$Q_v = 3.85 - 3.7$
$\gamma_{tr} = 5.034$
2rf, basic $h=2$, 160kV
t$_b$ = 400ns−100ns
at inj $\varepsilon \approx 200$ mm mrad

Bunch parameters (especially normalized)
similar to the heavy-ion bunches in SIS100 of FAIR
HEAD-TAIL INSTABILITY IN ISIS

Dedicated Experiment Campaign, 3 Shifts

$Q_v$ above 3.86:
Strong vertical oscillations and Losses

1rf example
$N_{\text{beam}} = 8.2 \times 10^{12}$

Growth rate generally difficult to determine

this example:
growth time $\tau = 0.1 \text{ms}$

Standing-wave pattern, exponential growth, wiggles due to $\xi$, $dQ/Q_s = 0.2$
$\Rightarrow k = 1$ head-tail mode
2rf example, flat bunch (length. mode)

\[ N_{\text{beam}} = 4 \times 10^{12} \]

Generally, long development difficult to observe, probably due to the small pipe/beam: early losses

2rf: usually complex mode structure

Here the \( k=1 \) mode as well?
2rf example, flat bunch (length. mode)
high intensity

$N_{\text{beam}} = 1.7 \times 10^{12}$

complex mode structure

growth time $\tau = 0.13 \text{ms}$
2rf example, Operational rf settings (note that \( Q_v \) not operational)

\[ N_{\text{beam}} = 14e12 \]

complex mode structure

here not discussed: during high-intensity operation at ISIS many different (and very interesting) modes observed at different Cycle Times
Try to use the approach of F. Sacherer 1974

\[ \Delta Q_k = \frac{\chi}{1 + k} \sum (-i) Z_{\perp} (\omega_p) h_k (\omega_p - \omega_\xi) \]
\[ \omega_p = (p + Q_0) \omega_0 + k \omega_a \]
\[ \chi = \frac{I_0 q_{\text{ion}}}{4 \pi \gamma mc Q_0 \omega_0} \]
\[ \omega_\xi = \frac{Q_0 \xi}{\eta} \omega_0 \]

Sacherer theory explains:
- Resistive-Wall Impedance as the drive: strong Z at small \((1-Q_{\text{frac}})\) \([Q_v \text{ above } 3.86]\)
- Higher \(k\) at later Cycle Times: higher \(\omega_\xi\)

Resistive-Wall Impedance unstable here

need to reduce the effective bunch length to 0.5 \(t_b\) in order to explain the \(k=1\) observation:
HEAD-TAIL INSTABILITY IN ISIS: THE DRIVE

\[ \Delta Q_k = \frac{\sum(-i)Z_{\perp}(\omega_p)h_k(\omega_p - \omega_\xi)}{1 + k\sum h_k(\omega_p - \omega_\xi)} \]

\[ \omega_p = (p + Q_0)\omega_0 + k\omega_s \]

\[ \text{Re}(Z_{\perp}^{rw}) = \frac{L_{rw}}{\pi b^3} \sqrt{\frac{cZ_0}{2\sigma_{rw}\Omega}} \]

Sacherer theory suggests:
- Reduced bunch length is needed to reproduce the $k=1$ mode
- Even with the overestimated Thick-Resistive-Wall Impedance the max growth rate corresponds to $\tau \approx 20\text{ms}$ (experiment $200\times$)

- Another Impedance with a similar $Z(f)$ in ISIS?
- Sacherer theory only partly adequate for these parameters?

Puzzles to solve in future
(Other approaches: G. Rees, RAL, 1992)
Focus of the present study:

**Intensity Thresholds** of the head-tail instability observed in ISIS

- **bottom thresholds**: instability and losses above this intensity
- **top thresholds**: no instability and losses above this intensity

Intriguing: repeatable top thresholds for very different bunches
HEAD-TAIL INSTABILITY: THRESHOLDS

\[ \gamma_{\text{MODE}} = \gamma_{\text{DRIVE}} + \gamma_{\text{DAMPING}} \]
\[ \gamma_{\text{DRIVE}} > 0 \]
\[ \gamma_{\text{DAMPING}} < 0 \]

Accelerator always has nonlinearities: produce damping and explains the **bottom thresholds**. Usually observed for coherent instabilities.

The DRIVE proportional to the intensity: \( \gamma_{\text{DRIVE}} = c Z N_{\text{beam}} \).
Bunch parameters do not change much.
The drive can not explain the top threshold.

There must be a non-linear enhancing DAMPING responsible for the **top thresholds**
Tune Shifts due to chromaticity, space-charge, nonlinearities, impedances.

For coasting beam works very well, dispersion relation D. Möhl (1969). Exception from the simple picture: space charge does not produce Landau damping of its own.

For head-tail modes in bunches more complicated:
Every mode $k$ has its own coherent shift;
Effect of space-charge is different;
Efficient coherent-incoherent interaction is different.
LANDAU DAMPING DUE TO SPACE-CHARGE

Landau damping in bunches exclusively due to the effect of space charge
[Burov PRSTAB 2009], [Balbekov 2009]

Our particle tracking simulations with the PATRIC code

\[ \Delta Q_{sc}(\tau) = \frac{\lambda(\tau) r_p R^2}{\gamma^3 \beta^2 Q_0 a^2} \]

space-charge tune shift

\[ q = \frac{\Delta Q_{sc}}{Q_s} \]

space-charge parameter

V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 114201 (2010)
HEAD-TAIL MODES WITH SPACE-CHARGE

The theory of an “airbag” bunch:
analytic prediction for arbitrary space charge
M.Blaskiewicz, PRSTAB 1, 044201 (1998)

\[ \Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2} \]

The theory perfectly reproduced in simulations (spectrum from PATRIC, dashed lines: theory):
V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 114201 (2010)
The Space-Charge tune shifts also observed and confirmed in experiment:
V.Kornilov, O.Boine-Frankenheim, PRSTAB 15, 114201 (2012)
Simulations confirm Landau damping in bunches exclusively due to the effect of space charge
V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 114201 (2010)
Landau Damping border $\Delta Q \approx -0.23 \Delta Q_{sc} + k Q_s$

Summary of Landau damping simulations:
region of Landau damping for each mode can be predicted;
strong space charge $q \gg 2k$ : no Landau damping for the mode.
SIMULATIONS: LANDAU DAMPING

Simulations so far: with the transverse K-V distribution. Transverse nonlinear Space-Charge increases tune spread and should enhance Landau damping.

Damping decrement from PATRIC simulations
HEAD-TAIL MODES WITH IMAGE CHARGES

include the effect of the image charges into the airbag theory:
O.Boine-Frankenheim, V.Kornilov, PRSTAB 12, 114201 (2009)

\[
\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}
\]

\[\Delta Q_{coh} = 0\]

\[\Delta Q_{coh} = 0.1 \Delta Q_{sc}\]

reproduced in simulations
LANDAU DAMPING DUE TO IMAGE CHARGES

\[ \Delta Q_k = -\Delta Q_{sc} + \Delta Q_{coh} = 0 \]

\[ \Delta Q_{coh} = 0.1 \Delta Q_{sc} \]

this should have an effect on Landau damping:

\[ \Delta Q_{coh} = 0 \]

\[ \Delta Q_{coh} = 0.1 \Delta Q_{sc} \]
LANDAU DAMPING DUE TO IMAGE CHARGES

\[ \Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2} \]

if we consider different image charge for the mode \( k=1 \):

- \( \Delta Q_{coh} = 0 \) (no damping)
- \( \Delta Q_{coh} = 0.1 \Delta Q_{sc} \)
- \( \Delta Q_{coh} = 0.2 \Delta Q_{sc} \)
Particle tracking simulations with PATRIC
$Q_s = 0.01$, Gaussian bunch longitudinally, transversally

- $\Delta Q_{coh} = 0.2 \, \Delta Q_{sc}$
- $\Delta Q_{coh} = 0.1 \, \Delta Q_{sc}$
- $\Delta Q_{coh} = 0$

basic agreement with the theory; detailed predictions of damping close to distribution tails are difficult.
Consider different strength of Space-Charge

Theory prediction:
keep the strength of space-charge fixed, increase the strength of imaga charges
Particle tracking simulations with PATRIC

$Q_s = 0.01$, Gaussian bunch longitudinally, transversally

$q = \Delta Q_{sc}/Q_s = 6$

$q = \Delta Q_{sc}/Q_s = 12$

basic agreement with the theory;
detailed predictions of damping close to distribution tails are difficult.
space-charge tune shift  \[ \Delta Q_{sc} = \frac{\lambda_0 r_p R}{\gamma^3 \beta^2 \varepsilon_{\perp}} \]

space-charge parameter  \[ q = \frac{\Delta Q_{sc}}{Q_s} \]

parameters of the 1RF bunch

beam 4e12p

beam 8.2e12p

the space-charge parameter \( q \) is steady during the cycle
Image Charges are an imaginary impedance and produce a coherent tune shift, proportional to the own space-charge tune shift:

$$\Delta Q_{coh} = \frac{\lambda_0 r_p R^2}{\gamma^3 \beta^2 Q_0} \frac{2 \xi_{h,v}}{h^2} = \Delta Q_{sc} \frac{a^2}{h^2}$$

$$\Delta Q_{sc} = \frac{\lambda_0 r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}$$

Round Pipe: $\xi_v = \xi_h = 0.5$

Rectangular Vacuum Chamber, centered beam (The book K.Y. Ng):

$$\xi_1^V = \frac{K^2(k)}{4} (1 - k')^2$$

$$\xi_1^H = K^2(k) k$$

the modulus $k$ calculated from the nome $q$

$$q = e^{-2\pi w/h}$$

half-height $h$, half-width $w$
Rectangular Vacuum Chamber
(The book K.Y. Ng)

\[ \xi_1^V = \frac{K^2(k)}{4} (1 - k)^2, \]
\[ \xi_1^H = K^2(k) k, \]

Calculation for ISIS:
\[ H = \langle h \rangle = 63.42 \text{mm} \]
\[ < 2\xi_h(s) / h(s)^2 > H^2 = 0.528 \]
\[ < 2\xi_v(s) / h(s)^2 > H^2 = 1.13 \]

results in vertical tune shift:
\[ \Delta Q_{\text{coh}} = 0.12 \Delta Q_{\text{sc}} \]
ISIS: charge-exchange multi-turn injection, painting in both planes (vertical sweep).

During these experiments, no exact transverse emittance for different intensity was available.

In order to check the model, three emittance(intensity) scenario considered: $k_N = 0, 0.5, 1$. 

\[
\Delta \varepsilon_\perp = k_N \frac{\Delta N_p}{N_{p0}} \varepsilon_{\perp 0}
\]

\[
\Delta Q_{\text{sc}} \propto \frac{N_p}{\varepsilon_\perp}
\]

\[
\frac{\Delta Q_{\text{coh}}}{\Delta Q_{\text{sc}}} \propto \varepsilon_\perp
\]
Predictions of the model:
Landau Damping borders of the mode $k=1$
for three different emittance (intensity)

$\Delta \varepsilon_\perp = k_N \frac{\Delta N_p}{N_{p0}} \varepsilon_{\perp 0}$

$\Delta Q_{sc} \propto \frac{N_p}{\varepsilon_\perp}$

$\Delta Q_{coh} \propto N_p$

$\frac{\Delta Q_{coh}}{\Delta Q_{sc}} \propto \varepsilon_\perp$

Intensity threshold for Landau damping
due to space-charge and image charges:
higher $\Delta Q_{coh} / \Delta Q_{sc}$
As the intensity increases, Landau Damping due to Space-Charge with Image-Charges boosts and can contribute to the top thresholds.

Additional observation at ISIS during this campaign, not mentioned so far: for the unstable bunch in 1RF, the transverse emittance was increased while keeping all the rest fixed: reproducible stability above \( \approx 90 \, \text{mm mrad} \).

Consistent with this Landau damping idea.

An emittance threshold.
CONCLUSIONS

Unstable head-tail modes observed in ISIS; beam parameter scans performed; the instability appearance suggests the **resistive-wall-like impedance** as the drive.

**Lower order** and a much **higher growth rate** than expected from the Sacherer theory. Probably bad news for SIS100 U-bunches, where k=4, τ≈100ms was expected.

Clear reproducible intensity thresholds found. Exceptional observation: intensity **top thresholds** (no instability above).

**Landau damping** due to **Space-Charge**, enhanced by the **Image-Charges**, obtained in **simulations**. Qualitative agreement with an airbag-bunch based model.

Calculations show that the unique large $\Delta Q_{coh}/\Delta Q_{sc}$ (**large beam/pipe**) ratio at ISIS should provide Landau damping due to Space-Charge, which can contribute to the top intensity thresholds, and emittance thresholds. Hence the recommendation: blow-up the beam (trade-off with other losses).

Calculations for SIS100 U-bunches ($\Delta Q_{sc}/Q_s$≈40, $\Delta Q_{coh}/\Delta Q_{sc}$≈0.08) indicate a possible effective Landau damping (**good news**).