IBS SUPPRESSION LATTICE IN RHIC: THEORY AND EXPERIMENTAL VERIFICATION


Abstract

Intra-beam scattering (IBS) is the limiting factor of the luminosity lifetime for Relativistic Heavy Ion Collider (RHIC) operation with heavy ions. Over the last few years the process of IBS was carefully studied in RHIC with dedicated IBS measurements and their comparison with the theoretical models. A new lattice was recently designed and implemented in RHIC to suppress transverse IBS growth, which lowered the average arc dispersion by about 20% [1]. This lattice became operational during RHIC Run-8. We review the IBS suppression mechanism, IBS measurements before and after the lattice change, and comparisons with predictions.

INTRODUCTION

IBS limits the present performance of the RHIC collider with heavy ions. To achieve required luminosities for a future upgrade of the RHIC complex, the Collider-Accelerator department at BNL has been developing several approaches to counteract IBS such as electron and stochastic cooling [2, 3]. Stochastic cooling in the longitudinal direction was already successfully implemented in RHIC, suppressing longitudinal emittance growth due to IBS. Implementation of stochastic cooling in the transverse plane is presently under development [3].

Simple features of IBS above transition energy allowed us to explore a possibility of reduction of the transverse IBS growth rate with a modification of the RHIC lattice [4]. This approach is less expensive and intrusive than cooling. The new RHIC lattice was developed over several dedicated Accelerator Physics Experiments (APEX). After successful experimental verification of the reduction of transverse IBS growth rate during APEX in June 2007, the new lattice became operational in the latest d-Au run in 2008.

In this paper, we describe the IBS-suppression mechanism, experimental measurements, and compare our measurements with predictions.

IBS IN RHIC

Particles within the beam scatter from one another via Coulomb collisions. When the scattering angles are small, the random addition of such small scattering events can lead to a growth of the beam dimensions. Such multiple Coulomb scattering was first applied to explain emittance growth in electron beams, and was called the "multiple Touschek effect" [5].

Piwinski later generalized multiple Coulomb scattering for proton machines [6], without approximations which were relevant for high-energy electron beams [5]. The generalized treatment of multiple small-angle Coulomb scattering was then renamed Intrabeam Scattering. IBS theory was later extended to include variations of the betatron functions and momentum dispersion function along the accelerator lattice, and was summarized by Martini (referred to here as “Martini’s model”) [7]. Bjorken and Mtingwa took a different approach to modeling IBS, using the scattering matrix formalism from quantum electrodynamics (“B-M model”) [8]. Both B-M and Martini’s models are in good agreement.

As far as Coulomb scattering is concerned, basic IBS scattering physics is the same as that of gas molecules in a closed box. However orbit curvature produces dispersion in circular accelerators. Because of this dispersion, an energy change leads to a change in the betatron amplitude. In other words, we have coupling of the longitudinal and transverse motion.

Another important consequence of the curvature in circular accelerators is that it leads to the so-called “negative mass” behavior of particles above transition energy. This in turn leads to different IBS consequences below and above transition energy, as pointed out by Piwinski [9]. Below transition energy the sum of horizontal, vertical and longitudinal oscillation amplitudes is bounded. Particles can only exchange energy, and behave like molecules in a closed box. However, above transition energy the total oscillation energy can increase. This leads to both longitudinal and transverse emittance growth.

Accurate expressions for the growth times of the beam dimensions are available [7, 8, 9]. We use these expressions [7-9] implemented in the BETACOOL code [10-11] for comparison with our experimental measurements. Approximate relations are given below to illustrate the IBS-suppression mechanism.

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For energies much higher than transition energy, longitudinal diffusion dominates IBS growth rates because the RMS velocity spread in the beam frame is much smaller in the longitudinal direction than in the transverse. Expressions for the growth rates can be significantly simplified in this case.

In a typical RHIC run, the beams are stored at energies ($\gamma \approx 10^7$) much higher than the transition energy ($\gamma_t \approx 23$). We therefore can use high-energy approximations for growth rates, when simple estimates are deemed sufficient. Dependence of the longitudinal growth rate on various beam parameters in the high-energy approximation is described, for example, by [12]:

$$\tau^{-1}_\parallel = \frac{1}{\sigma^2_p} \frac{d\sigma^2}{dt} \approx \frac{r_i^2 c N_i L_c}{8 \beta^3 \gamma^2 \epsilon_x^{1/2} \left( \frac{\beta_x^{1/2}}{\beta_x} \right) \sigma_p^2},$$

(1)

where $r_i$ is the classical radius of ion, $\sigma_p$ is the RMS momentum spread, $N_i$ is number of ions in the bunch, $L_c$ is the Coulomb logarithm, $\sigma_x$ is the RMS bunch length, $\beta_x \approx \beta$ is the average beta-function around the ring, and $\epsilon_x \approx \epsilon_y$ is the transverse beam emittance.

The transverse emittance growth rate is expressed in terms of the longitudinal growth rate as

$$\tau^{-1}_x = \frac{1}{\epsilon_x} \frac{d\epsilon_x}{dt} \approx \frac{\sigma^2_x}{\epsilon_x} \left( \frac{\sigma_x^2}{\beta_x} \right) \left( \frac{D_x^2 + (D_x' + \alpha_x D_x')^2}{\beta_x} \right) \tau^{-1}_\parallel,$$

(2)

where $\langle \rangle$ is an average around the ring, $\beta_x$ and $\alpha_x$ are the horizontal beta- and alpha-functions, and $D_x$ and $D_x'$ are the dispersion function and its derivative with respect to $s$. A similar expression holds for the vertical growth rate for lattices with vertical dispersion.

In RHIC, the vertical dispersion contribution averaged over the ring is close to zero. Longitudinal heating is thus transferred through the horizontal dispersion $D_x$ into the horizontal oscillation amplitude. For a lattice with transverse coupling, the horizontal growth rate due to IBS is shared between the horizontal and vertical directions. If the motion is completely coupled within a time period much smaller than IBS growth time, one can use average transverse rate for both degrees of freedom:

$$\frac{1}{\tau_{x,y,\text{coupled}}} = \frac{1}{2} \left( \frac{1}{\tau_x} + \frac{1}{\tau_y} \right)_{\text{uncoupled}},$$

(3)

which is the case for a typical RHIC run. In the general case without full coupling, IBS formalism for coupled motion is also available [9, 13].

**SUPPRESSION OF TRANSVERSE IBS AND LIMITATIONS**

The function in $\langle \rangle$ in Eq. (2) is the so-called $H$-function, which we denote as $H_x$. Another way to write the dispersion $H$-function is:

$$H(D_x, D_x') = \gamma_x D_x^2 + 2\alpha_x D_x D_x' + \beta_x D_x'^2.$$

(4)

The $H$-function is an invariant of the motion in dipole-free regions. Equation (2) immediately suggests that minimizing the $H$-function will reduce the transverse growth rate.

The RHIC lattice consists of six arcs and six interaction regions. Each arc consists of 11 FODO cells. The dominant $H_x$ contribution comes from the arcs. $H_x$ can be reduced by increasing the horizontal tune advance per arc FODO cell, $\Delta Q_x$. Due to a strong dependence of the $H$-function on the tune (in smooth approximation $H_x \approx 1/(Q_x)^2$), a small tune increase results in significant reduction of $H_x$. For example, an increase of the tune advance $\Delta Q_x$ per cell from 0.23 rad (82°) to 0.3 rad (107°) would result in a 2-fold reduction of $H_x$, strongly suppressing the transverse growth rate. This is the main idea behind the “IBS-suppression” lattice [14].

When the RHIC lattice was designed, IBS was one of the main concerns. This led to a lattice design with the shortest and strongest focusing FODO cells in the arcs than any other existing hadron collider. The choice of 82°-90° phase advance per cell was selected as an optimum between IBS and other effects [4, 15]. However, at the time of RHIC lattice design there were also many unknowns such as beam parameters, dynamic aperture, etc. For typical Au beam parameters in 2004, the 82° phase advance per cell lattice was not an optimum with respect to IBS. We then decided to proceed with lattice development with an incremental increase of the phase advance per cell.

The range of available tune advances per cell is limited by the existing RHIC power supplies and superconducting magnets. Also, since dispersion reduction requires larger sextupole strengths for chromaticity correction, dynamic aperture may become a limiting factor.

IBS-suppression lattice development started in 2005 with an increase of the horizontal phase advance per cell from 82° to 92° [16]. IBS growth rate simulations with expected beam parameters were performed using the BETACOOL code, which showed a 30% reduction in the horizontal emittance growth rate. This first step was considered to be sufficient to prove that the concept of IBS-suppression is correct. The lattice development and dedicated IBS measurements were done as part of Accelerator Physics Experiments (APEX).

Progress with the IBS-suppression lattice during the 2005 Cu-Cu run was limited. The main problems were related to the tune swings during the ramp and beam losses causing magnet quench protection interlocks. The 2006 polarized proton run could not be used for IBS-suppression lattice development. Progress with tune and coupling feedback in the 2007 Au-Au run lowered ramp development time with IBS-suppression lattice, which led to experimental verification of IBS-suppression concept in a June 2007 APEX experiment [1]. The IBS-suppression lattice with a 95° phase advance per cell was then implemented in operations for the Yellow RHIC ring in the 2008 d-Au run. We used the standard lattice in the Blue RHIC ring during this run, with 82° phase advance per cell, since IBS-suppression was not needed for the
deuteron beam. IBS is a much more important effect for heavy ions, as can be seen from Eq. (1) where IBS growth rate is proportional to the square of the classical radius of an ion, or \( \sim Z^4/A^2 \), where \( Z \) is the charge and \( A \) is atomic mass of the ion, respectively.

**IBS MODELS**

We used the accurate IBS models implemented in BETACOOL code for RHIC IBS growth rate prediction and benchmarking with experimental data. We used the corresponding design lattice, which includes the lattice function derivatives, for each analysis of APEX data [16].

The Martini and Bjorken-Mtingwa models were benchmarked vs. one another within the BETACOOL code for various lattice types and found to be in good agreement for RHIC parameters. These models are based on the assumption that beam distributions are approximately Gaussian. This assumption holds fairly well for the parameters of our dedicated experiments. A more accurate treatment of IBS is required for beam distributions which strongly deviate from Gaussian. A variety of models for treatment of IBS in non-Gaussian distributions, including the most recent model which treats amplitude-dependent diffusions coefficients in 3-D, are also available in BETACOOL. We use Martini’s model [7, 11] for our numerical simulations presented in this paper.

**PARAMETERS OF DEDICATED IBS MEASUREMENTS**

Over the last few years we performed a series of dedicated APEX IBS measurements to verify these IBS models. Machine parameters were configured to minimize or eliminate various effects which might complicate accurate comparison between the experimental data and the models. Good agreement between IBS models and measurements was found for both longitudinal and transverse emittance growth [17, 18].

We use RF with harmonic \( h=2520 \) and a total of about 3MV gap voltage in standard RHIC operations. The bunches fill the buckets already at the beginning of the store. Without recently implemented stochastic cooling there is then a significant beam loss from the bucket due to IBS. There is also the possibility of emittance growth driven by beam-beam collisions.

To insure accurate benchmarking of the IBS models, dedicated IBS measurements are taken with harmonic \( h=360 \) so the losses from the RF bucket are minimized, provided that the RF voltage is close to 300 kV. Beams are steered out of collision to avoid beam-beam contributions. In a typical experiment, we injected six bunches of different intensity and accelerated them to 100 GeV/nucleon beam energy in both rings. Different bunch intensities resulted in different emittances. This allowed us to verify the growth rate scaling with intensity and emittance in Eq. (1) and conclude that the scaling is consistent with IBS [18].

Longitudinal bunch length growth was measured for each individual bunch using wall current monitors. Horizontal and vertical emittances for each individual bunch were measured with ionization profile monitors (IPMs). The emittances were reconstructed from the measured RMS of the distributions and known beta-functions at the horizontal and vertical IPM locations.

**EXPERIMENTAL VERIFICATION**

**2007 APEX Experiment**

Figs. 1-3 show APEX IBS measurements for \( \gamma=107 \) Au ions in June 2007.

![Figure 1: Bunch intensity](image1)

![Figure 2: Bunch length](image2)

Beam Dynamics in High-Intensity Circular Machines
substantial intensity loss due to insufficient RF voltage during the measurements. At the time of the measurements, we had only 140kV per ring, compared to the typical settings of 300kV for dedicated IBS measurements.

In Fig. 2 one can see that bunches in the Yellow ring are shorter than in the Blue ring. This is a direct consequence of the new IBS-suppression lattice with higher phase advance per cell. Higher phase advance per cell results in higher tune and, correspondingly, in higher transition energy $\gamma_t$. For energies $\gamma >> \gamma_t$, the RMS bunch length $\sigma_s \sim 1/\sqrt{\gamma_t}$. The Blue ring with 82° lattice had $\gamma_t = 23$, while the Yellow 92° lattice had $\gamma_t = 26$. Expected reduction in bunch length in the Yellow ring is then 7%.

In Fig. 2, initial measured lengths of high-intensity bunches in the Yellow ring are 7-9% shorter than in the Blue ring. Also note that there was an initial intensity loss for bunches in the Blue ring while there was no such initial loss for bunches in the Yellow ring. This can be explained by the fact that RF bucket acceptance for the new lattice was increased by 14%, because of higher $\gamma_t$.

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For a direct comparison of IBS-induced beam growth between the two RHIC rings with different lattices, one must compare emittance growth vs. the corresponding “IBS-density” in the bunch. Here, this IBS-density is defined according to Eq. (1) as

$$n_{IBS}(t) = \frac{N_i(t)}{\sqrt{\langle \beta_x \rangle \epsilon_x(t) \epsilon_y(t) (\epsilon_x(t) + \epsilon_y(t)) \sigma_x(t)}}.$$

Data analysis shows that IBS-density is higher for bunches in the Yellow ring while the emittance growth rate in the Yellow ring is smaller than in the Blue ring. Due to very noisy data for the vertical emittance the error bars on the transverse emittance growth rates extracted from the data are rather large, but the analysis still shows that the horizontal emittance growth rate was reduced by 30±10% for the 92° lattice compared to the 82° lattice, in agreement with theoretically predicted 30% [1].

2008 APEX Experiment

The IBS-suppression lattice (dAu82) became operational in the RHIC Yellow ring during the 2008 d-Au run. This newly implemented lattice had 95° phase advance per arc cell and a transition gamma of $\gamma_t = 26.6$. We performed dedicated IBS measurements in the Yellow ring with $\gamma = 107$ Au ions in January 2008.

Figs. 4-5 show a comparison of measurements with BETACOOL simulations for one bunch. We show the time evolution of a bunch from RF bucket #121 for the case of a fully x-y coupled machine (fill #9552). Here we measured $\Delta Q_{\text{min}} = 0.018$, and separated the horizontal and vertical tunes by 0.018.

Figure 3: Horizontal emittances: 1) upper three curves – three high-intensity bunches in Blue ring; 2) lower three curves – three high-intensity bunches in Yellow ring.

Fig. 3 shows horizontal emittances of high-intensity bunches in the Blue and Yellow rings. Beam emittances are directly correlated with bunch intensities. From Fig. 1 one can see that bunches in the Yellow ring have lower intensities, leading to lower emittances in Fig. 3 compared to those in the Blue ring. We should note that vertical emittance measurements in the Yellow ring were very close to those of the horizontal emittance in Fig. 3; however, vertical emittance IPM measurements in the Blue ring were very noisy.

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Figure 4: Time evolution of the RMS bunch length: blue curve – simulations, red curve – measurements.

Figure 5: Time evolution of the 95% normalized horizontal emittance: red noisy curve – measurements; green dashed curve - simulations using 95° lattice; blue upper curve – simulations using 82° lattice.
Fig. 5 shows that the measured horizontal emittance growth (red noisy curve) is in good agreement with prediction (green dashed curve) for the 95° lattice. For the 82° lattice the corresponding growth rate would have been 30% larger, and one would expect to see the emittance evolution shown by the upper blue curve in Fig. 5. Fig. 6 shows the design dispersion functions for the two lattices which were used in the simulations that produced Fig. 5.

Table 1 summarizes the measured beam parameters with dAu82 lattice (95° phase advance, $\gamma_t=26.6$) for a bunch from RF bucket #121 at 620 sec after beginning of the store; these were used in simulations as initial parameters and no additional fitting was done.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$, $10^7$</td>
<td>0.92</td>
</tr>
<tr>
<td>$\epsilon_x$, $\mu$m (95%, normalized)</td>
<td>10.4</td>
</tr>
<tr>
<td>$\epsilon_y$, $\mu$m (95%, normalized)</td>
<td>10.3</td>
</tr>
<tr>
<td>RMS bunch length, cm</td>
<td>99</td>
</tr>
<tr>
<td>RMS momentum spread</td>
<td>0.00037</td>
</tr>
<tr>
<td>RF harmonic h</td>
<td>360</td>
</tr>
<tr>
<td>RF voltage, kV</td>
<td>300</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Figure 6: Design dispersion functions for two RHIC arcs for the 95° phase advance per FODO cell lattice (dAu82) and 82° lattice which were used in simulations for Fig. 5.

**CONCLUSIONS AND FUTURE PLANS**

The development of new IBS-suppression lattice for the RHIC collider confirmed reduction of transverse IBS growth rates with stronger arc focusing. IBS-driven emittance growth measured during dedicated experiments is in good agreement with theoretical predictions and computer simulations.

The IBS-suppression lattice offers a better vertex luminosity for operational conditions (197 MHz RF, rebucketing) due to shorter bunch length and larger RF bucket acceptance as a consequence of higher $\gamma_t$; this is in addition to luminosity improvement due to slower transverse emittance growth. The slower transverse emittance growth also permits us to achieve lower beta-functions at the collision points when beam lifetime is limited by this growth, also improving machine luminosity for physics.

Our next step is the development of a 107° FODO phase advance lattice which should provide a two-fold reduction in the transverse emittance growth rate compared to the standard 82° phase advance lattice [19].

**ACKNOWLEDGMENTS**

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**REFERENCES**