Statistical Theory of the SASE FEL Based on the Two-Particle Correlation Function Equation

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Outline

1. Introduction

2. Overview of the correlation function approach
   a. Particle motion equations
   b. Microscopic density distribution equation
   c. Correlation function equation

3. Application of the theory to particular cases
   a. Analytical solution for the case of cold beam at linear stage
   b. Numerical solution for the 1-D case and comparison with quasilinear approach

4. Conclusion
Statement of the Problem

System of $N$ particles with $6N$ discrete coordinates

Continuous distribution in single particle phase space

The problem has standard solution based on the BBGKY chain of equations. One has to apply it to the case of FEL.
Statement of the Problem

Single shot fluctuation

\[ I = I_0 + \delta I(z, t) \]

Averaging

\[ \langle I \rangle = I_0 \langle \delta I(z, t) \rangle = 0 \]

\[ \langle \delta I(z_1, t_1) \delta I(z_2, t_2) \rangle \neq 0 \]

Two-particle correlation function

\[ \langle \delta I(z_1, t_1) \delta I(z_2, t_2) \rangle \sim G_2(z_1, Q_1, t_1; z_2, Q_2, t_2) dQ_1 dQ_2 \]

\[ Q_i = (\Delta_i, x_i, x'_i, y_i, y'_i) \]
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   a. *Particle motion equations*
   b. *Microscopic density distribution equation*
   c. *Correlation function equation*

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4. Conclusion
a. Particle motion equations

Assumptions:

1. **Averaging over undulator period is possible** (all parameters vary slowly) – Fast oscillations are excluded

2. **Interaction is carried out through radiation field** - All other collective forces are neglected

3. **Radiation field has narrow bandwidth and obeys paraxial equation** - Explicit solution of the wave equation is used

4. **Interaction of particles and radiation field is resonant** - Expression for the interaction force is simplified

5. **Transverse trajectories of particles are prescribed** (transverse motion is not affected by FEL operation) – Only longitudinal motion equations are considered
\[ \frac{dz^{(k)}}{dt} = 1 - \frac{1}{2\gamma^2} + \frac{\Delta^{(k)}}{\gamma^2} - \Delta \beta \left(z^{(k)}, X^{(k)} \right) \]

\[ \frac{d\Delta^{(k)}}{dt} = \sum_{l \neq i} \Phi \left[z^{(k)}, X^{(k)}, z^{(l)}(t'^{(l)}), X^{(l)} \right] \]

\[ t - z^{(k)} = t'^{(l)} - z^{(l)}(t'^{(l)}) \]

**Energy deviation** \( \delta \gamma / \gamma_0 \) of the \( k \)-th particle

**Velocity shift due to betatron oscillations**

**Retardation**

**Longitudinal interaction “force” between two particles**

**Transverse trajectory**

\[
\Phi(1,2) = 2 \frac{r_k k_0 K(z_1)K(z_2)}{\gamma_0} \frac{\sin \left(k_w(z_1 - z_2) + k_0 \frac{\tilde{R}_1(X_1, z_1) - \tilde{R}_2(X_2, z_2)}{2(z_1 - z_2)} + \phi_1 - \phi_2 \right)}{z_1 - z_2} \theta(z_1 - z_2) \theta(z_2)
\]
Motion equations with retardation

System of ordinary differential equations

New "time" variable
\[ \Delta t = \Delta \xi + \Delta z \]

\[ \Delta \xi = \Delta t (1 - V_z) \]

\[ \frac{df}{d\xi} = \frac{df}{dt} \frac{1}{1 - V_z} \]

Derivative along the world line

**New independent variable is just a new parameterization of the electron world lines in the space-time continuum**
The light pulse is used to launch clocks.

New variable is similar to the zone time.
**b. Microscopic density distribution equation**

\[
N(z, \Delta, X; \theta) = \sum_k \delta(z - z^{(k)}(\theta)) \delta(\Delta - \Delta^{(k)}(\theta)) \delta(X - X^{(k)})
\]

\[
\left[ \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z_1} v_i + \int d\{2\} \Phi(1,2) N(2; \theta) \frac{\partial}{\partial \Delta_1} \right] N(1; \theta) = 0
\]

\[v_i = [1 + 2\Delta_i - 2\gamma_\parallel^2 \Delta \beta(z_i, X_i)]\]

\[(i) = (z_i, \Delta_i, X_i)\]

\[d\{i\} = dz_i d\Delta_i dX_i\]

Both distributions are taken at one moment of new “time”
Number of particles
in $\Delta z$ at given $\xi$

Transformation Law

$$N_{\theta}(z, \Delta, X; \theta) = (1 - V_z) N_t(z, \Delta, X; t)$$

$$N_z(t, \Delta, X; z) = V_z N_t(z, \Delta, X; t)$$

$\sim j = v \rho$

N - density of the world lines

$N(\Delta z, t_0) = 9$

$N(\Delta z, \xi_0) = 3$
c. Correlation function equation

Assumptions:

1. **Particles do not interact before they enter undulator** – One can determine initial distribution of coordinates at given $\xi$ from the distribution at the undulator entrance

2. **Particle initial coordinates for different shots are not correlated**
   – One can use zero initial condition for the correlation function
Physical Meaning of the Distribution Function

Probability to find the system of \( N \) particles in the \( 6 \times N \) dimensional phase space volume \( dX_1 \ldots dX_N \) at the “time” moment \( \theta \)

\[
f_N(1, \ldots, N; \theta)d\{1\} \ldots d\{N\} = f_N(X_1, \ldots, X_N; \theta)dX_1 \ldots dX_N
\]

Probability density

6D vector of one particle coordinates and momenta

\( m \)-particle distribution function

\[
f_m = \int f_N dX_{m+1} \ldots dX_N
\]

Probability to find one particle in the 6D phase space volume \( dX_1 \)

Probability to find one particle in the volume \( dX_1 \) and another particle in the volume \( dX_2 \)

\[
f_2(X_1, X_2; \theta)dX_1 dX_2
\]
Averaging of the microscopic distribution

\[
\langle N(1, \theta) \rangle = \int \sum_{i=1}^{N} \delta(X_1 - X^{(i)})f_N(X^{(1)}, \ldots, X^{(N)}; \theta) \, d\{X^{(1)}\} \ldots d\{X^{(N)}\} = Nf^{(1)}(1, \theta)
\]

\[
\langle N(1, \theta)N(2, \theta) \rangle = Nf_1(1, \theta)\delta(1 - 2) + N(N - 1)f_2(1,2, \theta)
\]

\[
V(i, j) = -N\Phi(z_i, X_i, z_j, X_j)\frac{\partial}{\partial \Delta_i}
\]

BBGKY chain of equations

\[
\left[ \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z_1} v_1 \right] f_1(1; \theta) = \int V(1,2) f_2(1,2; \theta) \, d\{2\},
\]

\[
\left[ \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z_1} v_1 + \frac{\partial}{\partial z_2} v_2 - \frac{1}{N} [V(1,2) + V(2,1)] \right] f_2(1,2; \theta) = \\
\int [V(1,3) + V(2,3)] f_3(1,2,3; \theta) \, d\{3\},
\]

\[
\left[ \frac{\partial}{\partial \theta} + \sum_{i=1}^{N} \frac{\partial}{\partial z_i} v_i - \frac{1}{N} \sum_{i \neq j}^{N} V(i, j) \right] f_N(1, \ldots, N; \theta) = 0,
\]

See e.g. S. Ishimaru, "Basic Principles of Plasma Physics".
Truncation of the BBGKY Chain

**Correlation functions decomposition**

\[
\begin{align*}
  f_1(1, \theta) &= F(1, \theta) \\
  f_2(1,2, \theta) &= F(1, \theta)F(2, \theta) + G(1,2, \theta) \\
  f_3(1,2,3, \theta) &= F(1, \theta)F(2, \theta)F(3, \theta) + F(1, \theta)G(2,3, \theta) + \\
  &\quad+ F(2, \theta)G(1,3, \theta) + F(3, \theta)G(1,2, \theta) + H(1,2,3, \theta)
\end{align*}
\]

\[
\begin{align*}
  f_n(1,\ldots,n, \theta) &= F(1, \theta) \cdots F(n, \theta) \\
  \Downarrow \\
  G(1,2,0) &= H(1,2,3,0) = \ldots = 0
\end{align*}
\]

We assume that \( H(1,2,3, \theta) \ll G(1,2, \theta) \)
Final System of Equations

\begin{align*}
\frac{\partial}{\partial \theta} F(1, \theta) + \frac{\partial}{\partial z_1} v_1 F(1, \theta) + N \frac{\partial F(1, \theta)}{\partial \Delta_1} & \int \Phi(1,2) F(2, \theta) d2 = -N \int \Phi(1,2) \frac{\partial G(1,2, \theta)}{\partial \Delta_1} d2 \\
\frac{\partial}{\partial \theta} G(1,2, \theta) + \frac{\partial}{\partial z_1} v_1 G(1,2, \theta) + v_2 G(1,2, \theta) + N \frac{\partial F(1, \theta)}{\partial \Delta_1} \int \Phi(1,3) G(2,3, \theta) d3 + \\
+ N \frac{\partial F(2, \theta)}{\partial \Delta_2} \int \Phi(2,3) G(1,3, \theta) d3 & = -\Phi(1,2) F(2) \frac{\partial F(1)}{\partial \Delta_1} - \Phi(2,1) F(1) \frac{\partial F(2)}{\partial \Delta_2}
\end{align*}

\text{shot noise induced source term}

v(i) = 1 + 2\Delta_i - 2\gamma_i^2 \left( \frac{x_i^2(z_i, X_i)}{2} + \frac{y_i^2(z_i, X_i)}{2} + \frac{x_i^2(z_i, X_i)}{2\beta_i^2(z_i)} + \frac{y_i^2(z_i, X_i)}{2\beta_i^2(z_i)} \right)

F(1,0) = F_0(z_1, \Delta_1), \ G(1,2,0) = 0

longitudinal “velocity” \ dz/d\theta

Initial conditions
Two-time Correlation Function

\[
\langle N(1, \theta_1)N(2, \theta_2) \rangle = N(N-1)f^{(2)}(1, \theta_1; 2, \theta_2) + NF_1(1, \theta_1, 2, \theta_2)
\]

\[
f^{(2)}(1, \theta_1; 2, \theta_2) = F(1, \theta_1)F(1, \theta_2) + G_2(1, \theta_1; 2, \theta_2)
\]

\[
\left( \frac{\partial}{\partial \theta_1} + \frac{\partial}{\partial z_1} \nu_1 \right)G_2(1, \theta_1; 2, \theta_2) = -N \frac{\partial F(1)}{\partial \Delta_1} \int \Phi(1,3)G_2(3, \theta_1; 2, \theta_2) dt\{3\}
\]

\[
G_2(1, \theta_1; 2, \theta_2) \big|_{\theta_1 = \theta_2} = G(1, 2, \theta_1)
\]

Initial conditions
For the coasting beam the problem becomes static

\[
\frac{\partial}{\partial z_1} v_1 F(1) + N \frac{\partial F(1)}{\partial \Delta} \int \Phi(1,2) F(2) d2 = -N \int \Phi(1,2) \frac{\partial G(1,2)}{\partial \Delta} d2
\]

\[
\frac{\partial}{\partial z_1} v_1 G(1,2) + \frac{\partial}{\partial z_2} v_2 G(1,2) + N \frac{\partial F(1)}{\partial \Delta} \int \Phi(1,3) G(2,3) d3 +
\]

\[
+ N \frac{\partial F(2)}{\partial \Delta} \int \Phi(2,3) G(1,3) d3 = -\Phi(1,2) F(2) \frac{\partial F(1)}{\partial \Delta} - \Phi(2,1) F(1) \frac{\partial F(2)}{\partial \Delta}
\]

Two-time correlation function depends only on \(\theta_1 - \theta_2\)

\[
\left( \frac{\partial}{\partial \theta_1} + \frac{\partial}{\partial z_1} v_1 \right) G_2(1,2, \theta_1 - \theta_2) = -N \frac{\partial F(1)}{\partial \Delta} \int \Phi(1,3) G_2(3,2, \theta_1 - \theta_2) d\{3\}
\]
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**a. Analytical solution for the case of cold beam at linear stage**

\[ F(\Delta, X) = \tilde{F}(X)\delta(\Delta) \]

\[ G(z_1, \Delta_1, X_1, z_2, \Delta_2, X_2) \]

Laplace transform

\[ G(s_1, \Delta_1, X_1, s_2, \Delta_2, X_2) \]

The cold beam distribution

\[ g_{mn}(s_1, r_1, s_2, r_2) = \int \int \int \Delta_1^{m} \Delta_2^{n} G(1,2) d\mathbf{r}_1 d\mathbf{r}_2 d\Delta_1 d\Delta_2 \]

Moments of correlation function

\[ g_{mn}(s_1, r_1, s_2, r_2) \]

Wide beam

\[ g_{mn}(s_1, s_2, \mathbf{r}_1 - \mathbf{r}_2) \]

Explicit solution

\[ g_{00}(s_1, s_2, k_\perp) = \frac{\chi}{N(s_1 + s_2)} \left\{ \frac{1 + 2N\chi s_1 \Phi(s_1, k_\perp) + s_2 \Phi(s_2, k_\perp)}{(s_1 + s_2)^2} \right\} - 1 \]

\[ D(s_1, s_2, k_\perp) = \left[ 1 + 2N\chi \frac{s_1 \Phi(s_1, k_\perp) + s_2 \Phi(s_2, k_\perp)}{(s_1 + s_2)^2} \right]^2 - 16(N\chi)^2 \frac{s_1 s_2 \Phi(s_1, k_\perp) \Phi(s_2, k_\perp)}{(s_1 + s_2)^4} \]
The gain length and the spectrum width are the same as in conventional theory.
Transverse Coherence Function

\[ g(z, \nu, r) = \frac{1}{(2\pi)^2} \int g(z, \nu, k_{\perp}) e^{ik_{\perp}r \cos(\varphi)} k_{\perp} dk_{\perp} d\varphi \sim \]

\[ \sim e^{\frac{z}{L_G}} \int e^{-\frac{1}{36L_G^2} \left( \nu + \frac{\sqrt{3}}{\pi} \right)^2} J_0 \left( x \frac{r}{\sqrt{L_G \lambda_0}} \right) x dx \]

\[ \nu = 0 \]

Inverse Fourier transform by \( k_{\perp} \)
**b. Numerical solution for the 1-D case and comparison with quasilinear approach**

**Quasilinear Equations**

\[ f(\tau, \Delta, z) = F_0(\Delta) + \sum_{\nu} f_{\nu}(\Delta, z) e^{i(1+\nu)\tau} \]

**Discrete Spectrum**

\[ \nu = H_\nu n \]

\[ \text{Signal is periodic in } \tau \]

\[ T = 2\pi / H_\nu \]

**Smoothed Single-Particle Phase Space Distribution**

\[ \frac{\partial F_0}{\partial z} = 2 \text{Re} \left( \sum_{\nu} \frac{\partial f_{\nu}}{\partial \Delta} \overline{A_\nu} \right) \]

\[ \frac{\partial f_{\nu}}{\partial z} + i(2\Delta - \nu) f_{\nu} = A_\nu \frac{\partial F_0}{\partial \Delta} \]

\[ \frac{dA_\nu}{dz} = 4\rho^3 \int f_{\nu}(\Delta, z) d\Delta \]
Initial Conditions for the Shot Noise

Macroscopic distribution of the initial shot noise

\[ f_0(\tau, \Delta) = \frac{T}{N_T} \sum_{i=1}^{N_T} \delta(\tau - \tau_i) \delta(\Delta - \Delta_i) \]

Fourier transform

\[ f_{0v}(\Delta) = \frac{1}{T} \int_0^T f_0(\tau, \Delta) e^{-i(1+v)\tau} d\tau = \frac{1}{N_T} \sum_{k=1}^{N_T} \delta(\Delta - \Delta_k) e^{-i\phi_k} \]

Smoothing of distribution

\[ f_{0v}(\Delta) = \frac{1}{N_T} \sum_m \Pi(\Delta - \Delta_m) \sum_{k=1}^{N_m} e^{-i\phi_k} \]

\[ \Pi(\Delta) = \frac{1}{H_\Delta} \text{ if } \Delta \in (-0.5H_\Delta, 0.5H_\Delta) \text{ and } \Pi(\Delta) = 0 \text{ otherwise} \]

Harmonics still contain delta-functions in energy

Random complex number \( X + iY \) with distribution

\[ P(X,Y)dXdY \approx \frac{1}{N\pi} e^{-\frac{1}{N}(x^2+y^2)} dXdY \]
\[ G_2(z_1, \Delta_1, z_2, \Delta_2, \theta_1 - \theta_2) \approx \sum_\nu \left\langle f_\nu(z_1, \Delta_1) f_\nu^*(z_2, \Delta_2) \right\rangle e^{i(1+\nu)(z_1 - z_2) - (\theta_1 - \theta_2)} \]

**Current Fluctuation Spectrum**

\[ J_\nu(z) = \int G_2(z, \Delta_1, z, \Delta_2, \tau) e^{i(1+\nu)\tau} d\Delta_1 d\Delta_2 d\tau \approx T \left\langle f_\nu(z, \Delta_1) f_\nu^*(z, \Delta_2) \right\rangle \]

**Current Fluctuation Power**

\[ J_0(z) = \int G(z, \Delta_1, z, \Delta_2) d\Delta_1 d\Delta_2 \approx \sum_\nu \left\langle f_\nu(z, \Delta_1) f_\nu^*(z, \Delta_2) \right\rangle d\Delta_1 d\Delta_2 \]
Results of Quasiliner Simulations

(blue – single run; red – averaged over 1000 runs)

Electron Efficiency

Current Fluctuation Power

Radiation Spectrum

Energy Distribution

Current Spectrum
Spectral Distributions at Different Points in Undulator

(step curve – single run; solid curve – averaged over 1000 runs)
Resulting System of Equations for Numerical Solution

One-time correlation function

\[ \nu_1 \frac{\partial}{\partial z_1} F(1) = -2 \text{Re} \left( \frac{\partial}{\partial \Delta_1} I(z_1, \Delta_1; z_1) \right) \]

\[ \frac{1}{2} \left[ (\nu_1 + \nu_2) \left( \frac{\partial}{\partial z_1} + \frac{\partial}{\partial z_2} \right) + (\nu_1 - \nu_2) \left( 2i + \frac{\partial}{\partial z_1} - \frac{\partial}{\partial z_2} \right) \right] \tilde{G}(1; 2) = \]

\[ = - \frac{\partial F(1)}{\partial \Delta_1} I^*(z_1; z_2, \Delta_2) - \frac{\partial F(2)}{\partial \Delta_2} I(z_2; z_1, \Delta_1) - \]

\[ - \frac{2\pi}{N_{\lambda_0}} \left( \tilde{\Phi}^*(z_1 - z_2) \frac{\partial}{\partial \Delta_1} + \tilde{\Phi}(z_2 - z_1) \frac{\partial}{\partial \Delta_2} \right) F(1)F(2) \]

\[ I(z_1; z_2, \Delta_2) = \int_0^{z_1} \int_{-\infty}^{\infty} \tilde{\Phi}(z_1 - z_3) \tilde{G}(2; 3) d\{3\} \]

\[ G(z_1, \Delta_1; z_2, \Delta_2) = 2 \text{Re} \left( \tilde{G}(z_1, \Delta_1; z_2, \Delta_2) e^{i(z_1 - z_2)} \right) \]

slow varying complex amplitude
Two-time correlation function

\[
\left( \frac{\partial}{\partial \tau} + (1 + 2\Delta) \frac{\partial}{\partial z} + 2i\Delta \right) \tilde{g}_2 = 0
\]

\[
= -\frac{\partial}{\partial \Delta} F(z, \Delta) \int \int \Phi^*(z - z') \tilde{g}_2(z', \Delta', \tau) d\Delta' dz'
\]

\[
\tilde{g}_2(z, \Delta, \tau) = \int \int \tilde{G}_2(z, \Delta, \tau; z_f, \Delta_2, 0) d\Delta_2
\]

\[
G_2(z_1, \Delta_1, \theta_1; z_2, \Delta_2, \theta_2) = 2 \text{Re} \left( \tilde{G}_2(z_1, \Delta_1, \theta_1; z_2, \Delta_2, \theta_2) e^{i(z_1 - z_2)} e^{-i(\theta_1 - \theta_2)} \right)
\]

slow varying complex amplitude
Algorithm of Numerical Solution

One-time correlation function

Explicit difference scheme

\[ G_{p_{j+1}}^{n,m} = G_{m_{j-1}}^{n,m} - \frac{\Delta_n - \Delta_m}{1 + \Delta_n + \Delta_m} \left( G_{p_{j-1}}^{n,m} - G_{m_{j+1}}^{n,m} + 4iH_z G_{o_{j+1}}^{n,m} \right) - \frac{2H_z}{H_{\Delta}} \frac{1}{1 + \Delta_n + \Delta_m} \left( (F_{n_{jo}}^{n} - F_{n_{jo}}^{n-1}) I1_{n_j}^{n} + (F_{n_{jo}}^{n} - F_{n_{jo}}^{n-1}) I2_{n_j}^{n} + \frac{2\pi}{N_{\lambda_0}} \frac{F_{n_{jo}}^{n} + F_{n_{jo}}^{n-1}}{2} (F_{n_{jo}}^{n} - F_{n_{jo}}^{n-1}) \tilde{\Phi}_{n_{jo-j}}^{n} \right) \]

\[ F_{j_{o+1}}^{n} = F_{j_{o-1}}^{n} - \frac{2}{1 + 2\Delta_n} H_z \frac{2H_z}{H_{\Delta}} \text{Re}(I2_{n_{jo}}^{n+1} - I2_{n_{jo}}^{n}) \]

\[ I1_{n_j}^{n} = \frac{H_z}{2} \sum_{k=1}^{j_{o-1}} (IG_{o_{j_{o}}^{m}}^{n,k} \tilde{\Phi}_{j_{o-k}+1}^{n} + IG_{o_{j_{o}}^{m}}^{n,k} \tilde{\Phi}_{j_{o-k}}^{n}) \]

\[ I2_{n_j}^{n} = \frac{H_z}{2} \sum_{k=1}^{j_{o-1}} (IG_{o_{j_{o}}^{n}}^{n,k} \tilde{\Phi}_{j_{o-k}+1}^{n} + IG_{o_{j_{o}}^{n}}^{n,k} \tilde{\Phi}_{j_{o-k}}^{n}) \]

\[ IG_{o_{j_{o}}^{n}}^{n} = H_{\Delta} \sum_{m=1}^{N_{\lambda}} G_{o_{j_{m}}^{n,m}} \]

\[ \tilde{G}(1,2) = \tilde{G}^{*}(2,1) \]
Algorithm of Numerical Solution

Two-time correlation function

Explicit difference scheme

\[
G_2 p_j^n = G_2 m_j^n - \frac{H_{\tau}}{H_z} (1 + 2\Delta_n) (G_2 m_j^n - G_2 m_{j-1}^n) - H_{\tau} 2i\Delta_n G_2 m_j^n - \frac{H_{\tau}}{H_{\Delta}} (F_j^n - F_j^{n-1})iG_2 m
\]

\[
IG2m = \frac{1}{2} H_z H_{\Delta} \sum_{k=1}^{j-1} \sum_{n} (G2 m_k^n \Phi_{j-k+1}^n + G2 m_{k+1}^n \Phi_{j-k}^n)
\]

\[
G_2\in_j^n = IG_0^{n}_{j, j+1}
\]
Results of Simulations Comparison

(blue – quasilinear approach; red – correlation function approach)

Current Fluctuation Power

<table>
<thead>
<tr>
<th>( J_{\nu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{\nu,\text{max}} = 7.1 \times 10^{-11} )</td>
</tr>
</tbody>
</table>

Energy Distribution

<table>
<thead>
<tr>
<th>( J_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z/L_g = 0.0 )</td>
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</tbody>
</table>

Current Spectrum

<table>
<thead>
<tr>
<th>( F(\Delta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \Delta \rho \rangle = 0 )</td>
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</table>
Energy and Spectral Distributions at Different Points in Undulator

(*dots* – quasilinear approach; *solid curve* – correlation function approach)
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Conclusion

1. We developed a new approach to the theory of SASE FELs based on correlation function equation.

2. New analytical and numerical results were obtained. Cross-checking with the quasilinear theory approach gives excellent mutual agreement.

3. The correlation function approach is significant for both right understanding of noise in FEL and obtaining of reliable calculation results. At this point only simplified model problems were solved this way, but the codes for real FEL calculations also look feasible.
Thank you for your attention!

The end.