IMPACT OF A CHIRP AND A CURVATURE IN THE ELECTRON ENERGY DISTRIBUTION ON A SEEDED FREE ELECTRON LASER

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Abstract

In an free electron laser, the electron beam entering the undulator can have an initial energy curvature besides an initial energy chirp. In this paper, we derive the FEL Green function for the case of the electron beam having both an energy chirp and an energy curvature by solving the coupled Vlasov-Maxwell equations. We give an integral representation as well as an analytic expression for the Green function, providing an analytical expression for its temporal duration and its bandwidth. The FEL radiation properties can be evinced directly by the green function when the seed laser is very longer or very shorter than the temporal duration of the green function: in this configurations analytical expressions of the shift and of the chirp of the FEL radiation frequency are provided.

INTRODUCTION

An x-ray free electron laser (FEL) calls for a high quality electron bunch with a low emittance, a high peak current and a high energy [1]. During the acceleration, bunch compression, and transportation, the electron bunch is subjected to the radio frequency (rf) curvature and wakefield effects. Thus, the energy profile of the electron bunch coming into the undulator can have temporal structure. These properties will impact the FEL process in the undulator. In this paper, we derive the Green function for the case when the electron bunch has initial energy chirp and energy curvature. With the Green function solved, the seeded FEL performance can be formulated as the seed convoluting with the Green function. Therefore, the impact of the electron bunch initial energy profile on the seeded FEL is ready to be studied. The effects of the electron bunch initial energy chirp on the FEL performance and possible short-pulse generation have been studied for Self-Amplified Spontaneous Emission (SASE) FEL [3, 4], and a seeded FEL as well [5, 6]. In the later case, the situation is complicated by the interplay of the electron energy chirp and a possible frequency chirp in the seed. In this paper, we further include the effect from a possible energy curvature in the electron bunch when it enters the undulator.

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VLASOV-MAXWELL ANALYSIS FOR AN INITIAL VALUE PROBLEM

To analyze the start-up of a seeded FEL amplifier we use the coupled set of Vlasov and Maxwell equations which describe the evolution of the electrons and the radiation fields [7]. Solving this set of equations provides an integral representation for the Green function.

Coupled Vlasov-Maxwell Equations

We follow the analysis and notation of Refs. [7, 3, 5]. Dimensionless variables are introduced as $Z = k_w z$, $\theta = (k_0 + k_w)z - \omega_0 t$, where $k_0 = 2\pi/\lambda_0$, $\omega_0 = k_0 c$, and $k_w = 2\pi/\lambda_w$ with λ_0 being the radiation wavelength, λ_w being the undulator period, and c being the speed of light in vacuum. We consider an electron bunch having both a linear chirp and a second-order curvature *i.e.*,

$$\gamma = \gamma_0 + \left.\frac{d\gamma}{dt}\right|_{t=0} t_0 + \frac{1}{2}\left.\frac{d^2\gamma}{dt^2}\right|_{t=0} t_0^2 + \cdots, \quad (1)$$

the electron distribution can be linearized on

$$\psi_0 = \delta(p + \mu\theta_0 + \nu\theta_0^2/2) \quad (2)$$

with γ the Lorentz factor of an electron in the electron bunch, $p = 2(\gamma - \gamma_0)/\gamma_0$ measure of energy deviation, $\theta_0 = \theta - pZ$, and γ_0 being the resonant energy.

$$\begin{cases} \mu = \left.\frac{d\gamma}{dt}\frac{2}{\gamma_0\omega_0}\right|_{t=0} \\ \nu = -\left.\frac{d^2\gamma}{dt^2}\frac{2}{\gamma_0\omega_0^2}\right|_{t=0} \end{cases}, \quad (3)$$

characterizing the energy chirp and the energy curvature in the electron bunch, respectively. Solving the Vlasov equation, and inserting the results in the Maxwell equation, assuming $\mu Z << 1$ and $\nu\theta Z << 1$ and considering only the seed evolution, (i.e. neglecting the SASE FEL term) we have:

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial\theta}\right) A(\theta, Z) = \quad (4)$$
$$i(2\rho)^3 \int_0^Z dZ'(Z - Z')e^{i(\mu\theta+\nu\theta^2/2)(Z-Z')} A(\theta, Z'),$$

where ρ is the Pierce parameter [8, 5]. Rewriting Eq. (4) in Laplace frequency domain $f(\theta, s) = \int_0^\infty dZ e^{-sZ} A(\theta, Z)$, we obtain the solution

$$f(\theta, s) = \int_{-\infty}^{\theta} d\theta' e^{-s(\theta-\theta')+K(\theta,\theta',s,\mu,\nu)} A(\theta', 0) \qquad (5)$$

where

$$K(\theta, \theta', s, \mu, \nu) = \int_{\theta'}^{\theta} \frac{i(2\rho)^3}{\left[s - i(\mu\theta'' + \nu\theta''^2/2)\right]^2} d\theta''. \qquad (6)$$

The inverse Laplace transform gives the FEL field envelope along the undulator. For convenience we introduce the following notation: $\hat{z} = 2\rho Z$, $\hat{s} = \rho\theta$, $\hat{\xi} = \rho\xi$, $\hat{\alpha} = -\mu/(2\rho^2)$, $\hat{\beta} = \nu/(2\rho^3)$, and $\hat{p} = s/(2\rho)$. After performing the contour integral first, the FEL field envelope can be written as the convolution of the initial seed with a Green function:

$$A(\hat{s}, \hat{z}) = \int_0^{\infty} d\hat{\xi} A\left(\hat{s} - \hat{\xi}, 0\right) g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right), \qquad (7)$$

with the Green function $g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right)$ defined as

$$g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) = 2 \int_c \frac{d\hat{p}}{2\pi i} e^{\hat{\mathcal{F}}\left(\hat{p}, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right)}, \qquad (8)$$

where $\hat{\mathcal{F}}$ denotes the phasor

$$\hat{\mathcal{F}}\left(\hat{p}, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) = \hat{p}(\hat{z} - 2\hat{\xi}) + \hat{K}\left(\hat{p}, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right). \qquad (9)$$

The exact form of $\hat{K}$ has been calculated:

$$\hat{K} = \left. \frac{2i\left[\frac{\hat{s}''\hat{\beta} - \hat{\alpha}}{i\hat{p} - \hat{s}''\hat{\alpha} + \hat{s}''^2\hat{\beta}/2} + \frac{2\hat{\beta}\arctan\left(\frac{\hat{s}''\hat{\beta} - \hat{\alpha}}{\sqrt{2i\hat{\beta}\hat{p} - \hat{\alpha}^2}}\right)}{\sqrt{2i\hat{\beta}\hat{p} - \hat{\alpha}^2}}\right]}{\hat{\alpha}^2 - 2i\hat{\beta}\hat{p}} \right|_{\hat{s}'' \to \rho\theta'}^{\hat{s}'' \to \rho\theta}. \qquad (10)$$

Notice that due to the energy chirp and energy curvature in the electron bunch, the Green function depends separately on ξ and θ, which are the FEL amplifier time variables, losing thus the translational invariance property. In general, for Green function without translation invariance, people normally decompose the Green function into parts which have translation invariance and parts which break translation invariance. The part which breaks translation invariance normally is small and treated as perturbation, which usually describes local effects.

Analytical approximation for the Green function

A closed analytical form for the Green function is estimated by a saddle point approximation. The saddle point $\hat{p}_s$ is found as the solution of $\left.\frac{d\hat{\mathcal{F}}(\hat{p})}{d\hat{p}}\right|_{p=p_s} = 0$ with $\hat{\mathcal{F}}$ given in Eq. (9) and $\hat{K}$ in Eq. (10). At the saddle point, the phasor has the highest real part, therefore, for long undulators, the growth mode with the highest real part dominates the other ones. The Green function is then approximated as

$$g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) \approx \frac{2\exp\left[\hat{\mathcal{F}}\left(\hat{p}_s, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right)\right]}{\left[2\pi\hat{\mathcal{F}}''\left(\hat{p}_s, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right)\right]^{1/2}}. \qquad (11)$$

As detailed in [2], to estimate $\hat{p}_s$, $\hat{\mathcal{F}}(\hat{p}_s)$ and $\hat{\mathcal{F}}''(\hat{p}_s)$ an order analysis is performed and the following expressions can be used to evaluate the Green function:

$$\begin{aligned}\hat{\mathcal{F}}(\hat{p}_s) &= i^{\frac{1}{3}}\hat{z} - \frac{i^{\frac{1}{3}}(\hat{z} - 6\hat{\xi})^2}{4\hat{z}} + \frac{i\hat{\alpha}}{2}(\hat{z} - 2\hat{\xi})(\hat{\xi} - 2\hat{s}) \\ &+ \frac{i\hat{\beta}}{6}(\hat{z} - 6\hat{\xi})(\hat{s} - \hat{\xi})^2 + \frac{i\hat{z}\hat{\beta}}{36}\Big(\frac{18\hat{\xi}\hat{z} - \hat{z}^2}{18} \\ &+ (\hat{s} - \hat{\xi})(\hat{z} - 6\hat{\xi} + 12\hat{s})\Big) + \frac{i^{\frac{5}{3}}\hat{z}^4}{432}\left(\frac{\hat{\alpha}^2}{\hat{z}} - \frac{\hat{\alpha}\hat{\beta}}{6} + \frac{\hat{z}\hat{\beta}^2}{135}\right) \\ &+ \frac{i^{-\frac{1}{3}}\hat{z}^2\hat{\beta}(\hat{s} - \hat{\xi})(12\hat{\alpha}\hat{\xi} - \hat{s}\hat{z}\hat{\beta})}{432} \\ &+ \frac{i^{-\frac{1}{3}}(\hat{z} - 6\hat{\xi})\hat{z}^3}{432}\left(\frac{\hat{\alpha}^2}{\hat{z}} - \frac{\hat{\alpha}\hat{\beta}}{3} + \frac{\hat{z}\hat{\beta}^2}{45} + \frac{\hat{\beta}^2(\hat{s} - \hat{\xi})}{6}\right)\end{aligned} \qquad (12)$$

$$\begin{aligned}\hat{\mathcal{F}}''(\hat{p}_s) &= \\ -\frac{i^{\frac{5}{3}}\left(5\hat{z}^2 - 24\hat{z}\hat{\xi} + 36\hat{\xi}^2\right)}{\hat{z}} &+ \frac{i\hat{z}^4}{108}\left(\frac{\hat{\alpha}^2}{\hat{z}} - \frac{\hat{\alpha}\hat{\beta}}{6} + \frac{\hat{z}\hat{\beta}^2}{135}\right) \\ + \frac{i\hat{z}^3\hat{\beta}(\hat{z} - 6\hat{\xi})}{648}\left(\hat{\alpha} - \frac{4\hat{z}\hat{\beta}}{45}\right) &+ \frac{i\hat{z}^3\hat{\beta}(\hat{s} - \hat{\xi})}{108}(\hat{s}\hat{\beta} - 2\hat{\alpha})\end{aligned} \qquad (13)$$

To further obtain a Green function that yields a closed form when convoluted with a Gaussian seed laser, we rewrite Eq. (11) as

$$g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) \approx \sqrt{\frac{2}{\pi}} e^{\hat{\mathcal{F}}\left(\hat{p}_s, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) - \frac{1}{2}\ln\left[\hat{\mathcal{F}}''\left(\hat{p}_s, \hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right)\right]} \qquad (14)$$

The expression of the exponential factor in Eq. (14), as detailed in [2] is given by

$$\begin{aligned}g\left(\hat{s}, \hat{z}, \hat{\xi}, \hat{\alpha}, \hat{\beta}\right) &\approx \frac{i^{\frac{1}{6}}}{\sqrt{\pi\hat{z}}}\exp\Bigg\{i^{\frac{1}{3}}\hat{z} - \frac{\hat{z} - 6\hat{\xi}}{2\hat{z}} - \frac{i^{\frac{1}{3}}(\hat{z} - 6\hat{\xi})^2}{4\hat{z}} \\ &+ \frac{i\hat{\alpha}}{2}(\hat{z} - 2\hat{\xi})(\hat{\xi} - 2\hat{s}) + \frac{i(6\hat{s} - \hat{z})(\hat{z} - 6\hat{\xi})\hat{\beta}}{216}(6\hat{s} + \hat{z} - 12\hat{\xi}) \\ &+ \frac{i\hat{z}\hat{\beta}}{36}\left[\frac{18\hat{\xi}\hat{z} - \hat{z}^2}{18} + (\hat{s} - \hat{\xi})(\hat{z} - 6\hat{\xi} + 12\hat{s})\right] \\ &- \frac{i^{\frac{4}{3}}\hat{z}^3}{432}\left(\frac{\hat{\alpha}^2}{\hat{z}} - \frac{\hat{\alpha}\hat{\beta}}{6} + \frac{\hat{z}\hat{\beta}^2}{135}\right)\left(1 - i^{\frac{1}{3}}\hat{z}\right) \\ &+ \frac{i^{\frac{4}{3}}\hat{z}^2\hat{\beta}(\hat{s} - \hat{\xi})}{432}\left[2\hat{\alpha} - \hat{\beta}\hat{s} - i^{\frac{1}{3}}(12\hat{\alpha}\hat{\xi} - \hat{\beta}\hat{s}\hat{z})\right] \\ &+ \frac{i^{\frac{4}{3}}\hat{z}^2(\hat{z} - 6\hat{\xi})}{432}\Bigg[\left(\frac{\hat{\alpha}^2}{\hat{z}} - \frac{\hat{\alpha}\hat{\beta}}{3} + \frac{\hat{z}\hat{\beta}^2}{45}\right)\left(1 - i^{\frac{1}{3}}\hat{z}\right) \\ &- \frac{\hat{\beta}^2(\hat{s} - \hat{\xi})}{6}\left(1 + i^{\frac{1}{3}}\hat{z}\right)\Bigg]\Bigg\}\end{aligned} \qquad (15)$$

PROPERTIES OF THE GREEN FUNCTION

The Green function contains a time independent contribution due to the undulator parameters and a time dependent contribution coming from the chirp and curvature on the electron bunch energy, thus it depends on both variables $\hat{\xi}$ and $\hat{s}$ separately. Both linear and quadratic terms have small influence on the position of the amplitude peak of the Green function, since $\hat{\alpha}$ and $\hat{\beta}$ only provides second order terms contributions. The terms $\hat{\alpha}$ and $\hat{\beta}$ affect much more the phase of the Green function as shown in Fig. 1.

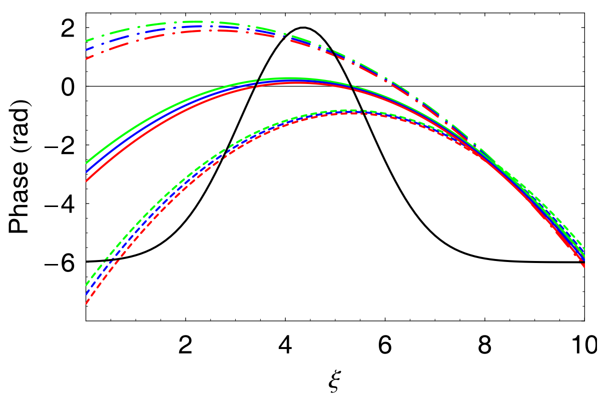

Figure 1: Phase of the green function vs $\hat{\xi}$ for $\hat{\alpha} < 0$ (dash dotted lines), $\hat{\alpha} = 0$ (solid lines) and $\hat{\alpha} > 0$ (dashed lines), varying $\hat{\beta}$ from negative (red), zero (blue) and positive (green) values. The solid black line corresponds to the module of the green function, which is negligible affected by $\hat{\alpha}$ and $\hat{\beta}$.

To evaluate the shift and the chirp of the FEL central frequency, we have to calculate respectively the first and the second derivative in $\hat{s}$ of the phase of Eq. (7). The ratio between the Green function temporal duration and the seed laser pulse temporal duration plays an important role. When the seed laser is longer than the Green function, the former is almost constant in the convolution of Eq. (7) and the FEL radiation phase is mostly given by the phase of the Green function amplitude peak. Thus the first and second derivatives on $\hat{s}$ of the FEL radiation phase are approximately given by the corresponding derivatives on $\hat{s}$ of the green function phase at its peak amplitude ($\hat{\xi} = \hat{\xi}_{Rpp}$), as shown in Fig. 2. The first derivative of the phase on $\hat{s}$ evaluated in correspondence of the maximum amplitude of the Green function and expanded to the first order of the chirps is

$$\left.\frac{\partial I}{\partial \hat{s}}\right|_{\hat{\xi}=\hat{\xi}_{Rpp}} \approx \frac{\sqrt{3}-3\hat{z}}{9}2\hat{\alpha} - \frac{\sqrt{3}(12\hat{s}+\hat{z})-3\hat{z}(12\hat{s}-\hat{z})-4}{54}\hat{\beta}. \quad (16)$$

Considering the order analysis between $\hat{\alpha}$, $\hat{\beta}$ and $\hat{z}$, Eq. (16) shows that even if the quadratic curvature has influence on the central frequency shift, the main effect comes from the linear chirp. The second derivative of the green

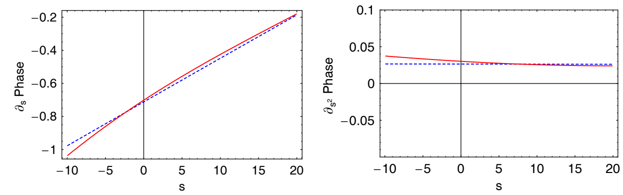

Figure 2: Comparison between the first (left) and the second (right) derivative of the integral in Eq. 7 (red solid line) for a long seed laser and the corresponding derivatives on $\hat{s}$ of the green function phase at its peak amplitude (blue dash line).

function phase on $\hat{s}$ has been calculated and its value at peak amplitude considering only the leading terms gives

$$\left.\frac{\partial^2 I}{\partial \hat{s}^2}\right|_{\hat{\xi}=\hat{\xi}_{Rpp}} \approx \frac{3z-\sqrt{3}}{9}2\hat{\beta} - z^2\frac{z-\sqrt{3}}{432}\hat{\beta}^2. \quad (17)$$

It is worthwhile to emphasize that regarding to the FEL radiation frequency chirp, the energy curvature of the electrons becomes the main source of the frequency chirp in case of long seed.

On the contrary when the seed laser is much shorter than the Green function (i.e. approaching to the delta Dirac function), the FEL radiation is given straightforwardly by the green function and deriving with respect to $\hat{s}$ equals to deriving with respect to $\hat{\xi}$ ($\partial\hat{\xi}/\partial\hat{s} \approx 1$). Thus to evaluate the shift of the FEL central frequency in this case, we consider the first derivative of the phase on $\hat{\xi}$. We evaluate it in the maximum amplitude of the Green function [*i.e.*, $\hat{\xi} = \hat{\xi}_{Rpp}$] choosing a delta function seed. Expanding to the first order in $\hat{\alpha}$ and $\hat{\beta}$ we obtain

$$\left.\frac{\partial I}{\partial \hat{\xi}}\right|_{\hat{\xi}=\hat{s}=\hat{\xi}_{Rpp}} \approx -\frac{\sqrt{3}}{\hat{z}} + \frac{\hat{z}}{2}\hat{\alpha} - \frac{\hat{z}^2}{36}\hat{\beta} + \frac{6-\sqrt{3}\hat{z}}{54}\hat{\beta} \quad (18)$$

The second derivative of the phase on $\hat{\xi}$ gives the radiation frequency chirp in case of a short seed:

$$\left.\frac{\partial^2 I}{\partial \hat{\xi}^2}\right|_{\hat{s}=\hat{\xi}_{Rpp}} \approx -\frac{9}{\hat{z}} - 2\hat{\alpha} + \left(\frac{4}{3\sqrt{3}} + \frac{\hat{z}}{3}\right)\hat{\beta} \quad (19)$$

In this configuration Eq. (19) shows that both linear and quadratic chirp have influence on the frequency chirp but the main effect is due to the linear component. The first term is the intrinsic frequency chirp due to the FEL interaction [5].

Similarly to [5] we calculate the temporal duration and the bandwidth of the Green function:

$$\sigma^2_{t,\hat{\alpha},\hat{\beta}} = \frac{k_w z}{9\sqrt{3}\rho\omega_0^2}\frac{1}{1-2\eta}, \quad (20)$$

$$\sigma^2_{\omega,\hat{\alpha},\hat{\beta}} = \frac{3\sqrt{3}\rho\omega_0^2}{k_w z}\left(1 + \frac{\kappa+\kappa^2+2\kappa\eta+4\eta^2}{1-2\eta}\right), \quad (21)$$

where

$$\eta = \frac{k_w^3 z^3 \hat{\beta} \rho^3 \left[-36\hat{\alpha} + \hat{\beta}\left(\sqrt{3} + 6k_w z\rho\right)\right]}{2916}, \quad (22)$$

$$\kappa = \frac{2}{9}k_w z\hat{\alpha}\rho - \frac{10}{27}k_w^2 z^2 \hat{\beta}\rho^2 + \frac{k_w^3 z^3 \hat{\beta}^2 \rho^3}{486\sqrt{3}} - \frac{4}{9}k_w z\hat{\beta}\rho^2 (k_0 z - \omega_0 t). \quad (23)$$

Eq. (20) reveals that the rms temporal duration is affected through the parameter η only by the energy curvature in the electron bunch, and this is a second order effect. The Green function bandwidth in Eq (21) can be further simplified neglecting 2η compared to 1 and third order quantities in $\hat{\alpha}$ and $\hat{\beta}$ obtaining the following approximated expression:

$$\sigma^2_{\omega,\hat{\alpha},\hat{\beta}} = \frac{3\sqrt{3}\rho\omega_0^2}{k_w z}\left(1 + \kappa + \kappa^2\right). \quad (24)$$

The FEL pulse travels with a group velocity [5] of $v_g = \omega_0/(k_0 + 2k_w/3)$, we have $k_0 z - \omega_0 t = -2k_w z/3$. The complete expression of the group velocity as function of $\hat{\alpha}$ and $\hat{\beta}$ can be well approximated with the above expression since the terms in $\hat{\alpha}$ and in $\hat{\beta}$ are at least of the second order. So, the expression of κ in Eq. (22) can be simplified as

$$\kappa = \frac{2}{9}k_w z\hat{\alpha}\rho - \frac{2}{27}k_w^2 z^2 \hat{\beta}\rho^2 + \frac{k_w^3 z^3 \hat{\beta}^2 \rho^3}{486\sqrt{3}}. \quad (25)$$

The variation of the bandwidth with respect to the unchirped case is plotted in figure 3 as function of $\hat{\alpha}$, for $\hat{\beta}$ assuming negative, zero and positive values. As concrete example Fig. 4 shows the longitudinal phase space of a typical bunch at 1.2GeV for the FERMI@elettra project [9]. The obtained $\hat{\alpha}$ and $\hat{\beta}$ change the green function bandwidth of about 3%.

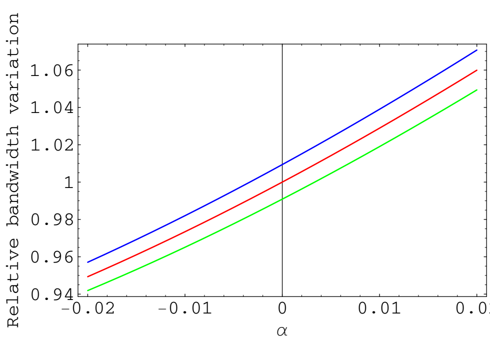

Figure 3: Relative variation of $\sigma_{\omega,\hat{\alpha},\hat{\beta}}$ vs $\hat{\alpha}$ for $\hat{\beta} > 0$ (green), $\hat{\beta} = 0$ (red) and $\hat{\beta} < 0$ (blue).

CONCLUSION

The FEL Green function for the case of the electron bunch having both an energy chirp and an energy curvature has been derived by solving the coupled Vlasov-Maxwell equations. The study of the obtained Green function has revealed that the linear term of the electron energy chirp provides a shift in the FEL radiation frequency and it is responsible for the radiation frequency chirp when the seed laser is much shorter than the Green function temporal duration, and close to a Dirac delta-function. Otherwise in case of longer seed laser the energy curvature is the main responsible for the frequency chirp of the FEL radiation. Moreover the energy curvature in the electron bunch influences the Green function rms temporal duration. An approximated expression of the Green function bandwidth as function of $\hat{\alpha}$ and $\hat{\beta}$ was found.

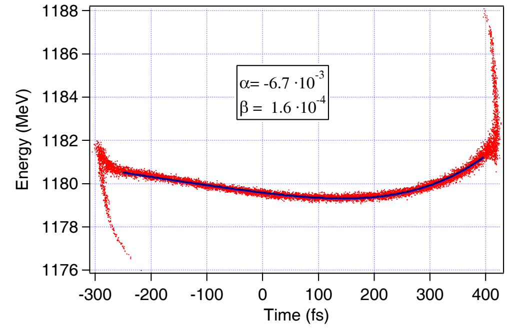

Figure 4: Longitudinal phase space of a 1.2GeV bunch simulated for the FERMI@elettra project.

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