Abstract

This paper gives a brief overview of various beam and spin dynamics investigations undertaken in the framework of the design studies regarding the FFAG lattice based electron energy recovery recirculator ring of the eRHIC electron-ion collider project.

INTRODUCTION

A Fixed Field Alternating Gradient (FFAG) doublet-cell version of the energy recovery recirculator of the eRHIC electron-ion collider [1] is being investigated [2, 3]. A pair of such FFAG rings placed along RHIC recirculate the electron beam through a 1.322 GeV linac (ERL), from respectively 1.3 to 6.6 GeV (5 beams) and 7.9 to 21.2 GeV (11 beams), and back down to injection energy. A spreader and a combiner are placed at the linac ends for proper orbit and 6-D matching, including time-of-flight adjustment.

FFAG LATTICE

The second, 11 beam, 21.2 GeV ring is considered in this discussion since it produces the major SR induced particle and spin dynamics perturbations. The cell is shown in Fig. 1, there are 138 such cells in each one of the 6 eRHIC arcs. The 6 long straight sections (LSS) use that very cell, with quadrupole axes aligned. In the twelve, 17-cell, dispersion suppressors (DS) the quadrupole axes slowly shift from their distance in the arc, to zero at the LSS.

Figure 2 shows the transverse excursion and magnetic field along orbits across the arc cell. Figure 3 shows the energy dependence of the deviation angle and curvature radius in the two quadrupoles, and the energy dependent tunes and chromaticities.

A Note on Dispersion Suppressors

The 12 dispersion suppressors are based on a “missing bend” scheme, where the relative displacement of the two cell quadrupoles (the origin of the dipole effect in the FFAG cell) is brought to zero over a series of cells. From orbit viewpoint, a quadrupole displacement is equivalent to a kick $\theta_k$ at entrance and exit [4] (see appendix).

Upon equivalent defect kicks due to the varying displacement of the quads (from their misalignment in the arc to aligned configuration in the straight) the orbit builds along the DS (with origin at upstream arc, end at downstream straight, or reverse) following

$$x_{orb}(s) = x_{orb}(0) \frac{\cos(\phi)}{\sqrt{\beta(0)}} + \alpha(0) x_{orb}(0) + \frac{\beta(0)}{\sqrt{\beta(0)}} \sin(\phi) + \sum_k \sqrt{\beta(s_k)} \theta_k \sin(\phi - \phi_k)$$

$$\frac{\alpha(s) x_{orb}(s) + \beta(s) x'_{orb}(s)}{\sqrt{\beta(s)}} = -\frac{x_{orb}(0)}{\sqrt{\beta(0)}} \sin(\phi) + \alpha(0) x_{orb}(0) + \frac{\beta(0)}{\sqrt{\beta(0)}} \sin(\phi) +$$

Figure 1: Arc cell in the 7.944-21.16 GeV recirculating ring.

Figure 2: Transverse excursion in the quadrupole frame (hence artefact of trajectory discontinuity) (top) and hard-edged magnetic field (bottom) along the 11 orbits across the arc FFAG cell.

Figure 3: The $y$-precession of the spin over the six 138-cell arcs amounts to $6 \times 138 \times a \gamma \theta_{cell} = a \gamma \times (2\pi - 0.688734)$ rad (with the difference to $a \gamma \times 2\pi$ corresponding to the contribution of the 12 DS), i.e., from 18 precessions at 7.944 GeV to 48 at 21.164 GeV. ($a = 0.00116$ is the electron anomalous magnetic factor, $\gamma$ the Lorentz relativistic factor).

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with \( x_{\text{orb}}(0) \) and \( x'_{\text{orb}}(0) \) the FFAG orbit coordinates in the arc→LSS case, while \( x_{\text{orb}}(0) = 0, x'_{\text{orb}}(0) = 0 \) in the LSS→arc case. Figure 4-top shows the orbit build-up from LLS to arc, ending up at the arc with \((x, x')\) coordinates which do not fully coincide with the periodic orbit of the arc FFAG cell. The orbit build-up depends on the phase advance \( \phi = \int_0^s \frac{ds}{\\beta(s)} \), as a consequence it depends on cell tune, and thus on energy. Figure 4-middle shows the resulting orbit build-up in the arcs over 6 consecutive arcs at 5 different energies, 7.9, 9.3, 10.6, 11.9 GeV and 13.2 GeV. In each case the starting coordinates (at \( s = 0 \) in the figure, i.e., in the first LSS) are taken \((x, x') = (0, 0)\). Figure 4-bottom illustrates the tune dependence of the orbit amplification in the case of pass #4 - for simplicity energy is changed instead of tunes, with the correlation given in Fig. 3.

### SYNCHROTRON RADIATION

The SR induced energy loss relative to the bunch centroid and the energy spread write, respectively

\[
\frac{\Delta E}{E_{\text{ref}}} = 1.9 \times 10^{-15} \frac{\gamma^3 \Delta \theta}{\rho}, \quad \sigma_E \approx 3.8 \times 10^{-14} \frac{\gamma^2 \sqrt{\Delta \theta}}{\rho} (2)
\]

with \( \Delta \theta \) the arc length and \( 1/\rho \) the curvature, assumed constant. Taking for average radius, in the QF (focusing quad) and BD (defocusing quad) magnets respectively, \( \rho_{\text{BD}} \approx \frac{\ell_{\text{BD}}}{\Delta \theta_{\text{BD}}}, \rho_{\text{QF}} \approx \frac{s_{\text{QF}}}{\Delta \theta_{\text{QF}}} \) (with \( s_{\text{BD}} \) and \( s_{\text{QF}} \) the arc lengths) and considering in addition, with \( l_{\text{BD}}, l_{\text{QF}} \) the magnet lengths, \( s_{\text{BD}} \approx l_{\text{BD}}, s_{\text{QF}} \approx l_{\text{QF}} \), and taking in addition \( (1/\rho)^2 \approx 1/\rho^2 \), then one gets, per cell

\[
\Delta E [\text{MeV}] \approx 0.96 \times 10^{-15} \frac{\gamma^4}{\rho_{\text{BD}}^2 + \rho_{\text{QF}}^2} \quad \sigma_E \approx 1.94 \times 10^{-14} \frac{\gamma^2}{\sqrt{\rho_{\text{BD}}^4 + \rho_{\text{QF}}^4}} (3)
\]
This is illustrated for a complete eRHIC turn (including LSS and DS sections) in Fig. 5, where it is also compared with Monte Carlo tracking, the agreement is at % level. The energy loss shows a local minimum in the $\sigma_\gamma = 30 - 35$ region, a different behavior from the classical $\gamma^4$ dependence in an isomagnitic lattice.

![Energy loss and energy spread](image)

Figure 5: Energy loss and energy spread. Solid lines: theory (Eqs. 3) for a 6-arc ring. Markers: Monte Carlo, for a complete eRHIC ring (see sample tracking outcomes in Fig. 6).

The bunch lengthening over a $[s, s_f]$ distance, resulting from the stochastic energy loss, can be written \[ \sigma_l = \left( \frac{\sigma_E}{E} \right) \left[ \frac{1}{L_{s\text{bend}}} \int_{s}^{s_f} (D_x(s)T_{51}(s_f \leftarrow s) + (4)

\right. \left. D'_x(s)T_{52}(s_f \leftarrow s) - T_{56})^2 ds \right]^{1/2} \]

with the integral being taken over the bends, $D_x$ and $D'_x$ the dispersion function and its derivative, $T_{5i}$ the trajectory lengthening coefficient of the first order mapping ($i = 1, 5, 6$ stand for respectively $x$, $\delta l$, $\delta p/p$ coordinates).

The energy loss causes a drift of the bunch centroid, as well as an horizontal emittance increase, both can be computed from the lattice parameters in the linear approximation [5, 6, 7]. Figure 7 illustrates these effects over a 21.164 GeV recirculation (with bunch re-centering on the reference optical axis at each of the six LSS).

Cumulative effect of SR, over a complete 7.94→21.2→7.94 GeV cycle, is illustrated in Fig. 8: (i) energy spread, $\sigma_E/E = 2.6 \times 10^{-4}$ at 21.1 GeV and $\sigma_E/E = 8.4 \times 10^{-4}$ back at 7.944 GeV; (ii) bunch lengthening, $\sigma_l = 2$ mm at 21.1 GeV and $\sigma_l = 2.5$ mm back down to 7.944 GeV; (iii) normalized horizontal emittance (from zero starting value), namely, $\epsilon_x = 20 \mu m$ at 21.1 GeV (with strong contribution from uncompensated chromatic effects), and $\epsilon_x = 8 \mu m$ back at 7.944 GeV.

Acceptance

The naturally large dynamical acceptance of the linear lattice shrinks with magnet alignment and field defects, this is illustrated in Fig. 9. SR is off in these DA computations, SR causes emittance growth thus reducing the space available for the beam at injection into a recirculation.

![Acceptance](image)

Figure 6: Top: stochastic energy decrease of a few particles over the first 3 arcs at $E_{\text{ref}} = 21.164$ GeV. Middle: final spread a 5000 particle bunch after the 21.164 GeV pass, $\frac{\Delta E_{\text{ref}}}{E_{\text{ref}}} = 1.9 \times 10^{-4}$ around $\frac{\Delta E_{\text{ref}}}{E_{\text{ref}}} = -4.7 \times 10^{-3}$ average energy loss (Eq. 2). Bottom: longitudinal bunch distribution (Eq. 4).

![Multipole Defects](image)

Figure 7: Left: SR loss induced x-drift along the 6 arcs, complete ring, $E = 21.164$ GeV, (shown are a few particles in a bunch launched on the LSS axis with zero initial 6-D emittance). Right: a 5000 particle bunch, horizontal phase space after that complete turn, featuring $\epsilon_x = 15 \mu m$, $\sigma_{\epsilon_x} = 4.3 \mu m$, $\sigma_{\epsilon'_x} = -1.1 \mu m$, $\sigma_{\epsilon'_x} = 1.8 \mu m$.

Multipole Defects

Figure 10 illustrates a different way of looking at tolerances, e.g. here in the presence of a dodecapole defect in all quadrupoles of the ring (i.e., same working hypotheses as for the bottom Fig. 9): a 5000-particle bunch is launched with $\epsilon_x = \epsilon_y = 50 \pi \mu m$ and $10^{-4}$ rms energy spread, for 21 circulations in a complete ring ($6 \times \left[ \frac{1}{2}\text{LSS} - \text{DS} - \text{ARC} - \text{DS} - \frac{1}{2}\text{LSS} \right] + \text{Linac}$).
Polarized electron bunch production is based on a Gatling gun, with a polarization of 85-90%. The electron bunch is re-circulated in eRHIC with longitudinal polarization. Spins precess at a rate $a\gamma$ per turn, with an increment of $a\Delta\gamma = 3$ at each 1.322 GeV linac boost, so ensuring the requested longitudinal spin orientation at the two IPs.

Depolarization mainly stems from energy spread (e.g., a cumulated $2.5 \times 10^{-4}$ at 21.2 GeV from SR contribution, see Fig. 8). Spin diffusion resulting from stochastic SR also causes polarization loss, of about 2% at 21.2 GeV. Non-zero vertical emittance, or vertical defects, cause spins to leave the median plane. This is illustrated in Fig. 11.
Figure 12 monitors the evolution of the polarization and of spin angle spreading, in the conditions of dodecapole error simulations discussed earlier (“Multipole defect” section and Fig. 10). Both quantities appear unchanged in this particular case, compared to the unperturbed optics (cf. $\sigma_\phi$ in Fig. 11-left).

![Graph showing polarization and spin angle spreading](image)

Figure 12: Polarization (right vertical axis) and spin angle spread (left axis) in the presence of dodecapole errors.

**CHROMATIC EFFECTS**

Due to the large chromaticity (Fig. 3), any beam misalignment results in phase extent in phase space according to $\Delta \phi = 2 \pi \xi dE/E$. SR is an intrinsic cause since it introduces both energy spread and beam shift [9], its effect is small however compared to nominal beam emittances, it is illustrated in Fig. 13 which shows the phase-space portrait acquired by a bunch launched with zero emittances and energy spread, after a single 11 GeV pass in the eRHIC ring, assuming a sub-millimeter beam misalignment in the DS regions. Note that here we introduce a measure (used in the following) of that chromaticity related effect in terms of the rms emittance, namely, surface in phase space $\epsilon_x = 4 \pi \sqrt{< x^2 > < x'^2 > - < xx' >^2}$ (same for $(y,y')$ space), which is thus an apparent emittance, including momentum spread induced surface increase.

Since the chromaticity is not corrected in the eRHIC linear FFAG lattice, and given the natural beam energy spread $\sigma E/E$ in the $2 \times 10^{-4}$ range, thus the emittance growth is prohibitive in the absence of orbit correction. This is illustrated, for the horizontal motion, in Fig. 14 which shows the phase space portraits of a 5000-particle bunch at the end of pass 11 (21.2 GeV, collision energy), and at the end of pass 21 (back to 7.944 GeV), whereas initial conditions at start, 7.944 GeV, were Gaussian $\epsilon_x \approx \epsilon_y \approx 50 \pi \mu$m and $dE/E = 0$. Top: end of the 21.2 GeV pass (collision energy), bottom: end of the the decelerated 7.9 GeV last pass.

![Graph showing horizontal phase space portraits](image)

Figure 14: Horizontal phase space portrait of a bunch launched at 7.944 GeV with initial Gaussian rms $\epsilon_x \approx \epsilon_y \approx 50 \pi \mu$m and $dE/E = 0$. Top: end of the 21.2 GeV pass (collision energy), bottom: end of the the decelerated 7.9 GeV last pass.

Fig. 15 summarizes the overall apparent emittance increase, over the 11 accelerated passes (from 7.944 to 21.16 GeV) followed by 10 decelerated passes (from 21.16 back to 7.944 GeV), for a bunch launched at 7.944 GeV with initial Gaussian $\epsilon_x \approx \epsilon_y \approx 50 \pi \mu$m and $dE/E \in [-10^{-4}, +10^{-4}]$ (random uniform).

Figure 16 shows the much reduced emittance growth in the accelerating phase and $\alpha \gamma \approx 38$, 28 on the decelerating phase.

Figure 17 is obtained in the case of a vertical orbit defect caused by a small dipole error $a_0 \in [-1, +1]$Gauss, random uniform, injected in all the quadrupoles of the ring. The bunch in this case is recentered at the linac, in both transverse planes, at each turn.

Figure 18 displays the evolution of the polarization (the projection, $\cos(\Delta \phi)$, of the 5000 spins on the average spin direction) and of the spin angle spread $\sigma_\phi$, in the previ-
CLOSED ORBIT DEFECTS

Dipolar type of errors due to magnet misalignments and dipole field defects, can be approximated by pairs of identical entrance/exit kicks [4], recalled in Tab. 1, such that...
\[ \theta_{en}/\theta_{ex} = \Delta(B_l)/B_\rho, \text{ with } \Delta(B_l) \text{ representing the effect of the imperfection.} \]

### Table 1: Defect Equivalent Closed Orbit Kicks

<table>
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<th>Formulas(^{(a)})</th>
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<tr>
<td><strong>Horizontal c.o.</strong></td>
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<tr>
<td><strong>Dipole H kicks</strong></td>
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<tr>
<td>(\theta/\delta L/L) &amp; (-\theta/(2\cos(\theta/2)))</td>
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<tr>
<td>(\theta/\delta B/B) &amp; (-\tan(\theta/2))</td>
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<tr>
<td>(\theta/\delta x) &amp; (\sin(\theta/2 - \alpha)/(\rho\cos(\alpha)))</td>
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<tr>
<td>(\theta/\phi_s) &amp; (\pm\cos(\theta/2 - \alpha)/(\rho\cos(\alpha)))</td>
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<tr>
<td>(\theta/\phi_x) &amp; (\mp\sin(\theta/2)\sin(\theta/2 - \alpha)/\cos(\alpha))</td>
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<tr>
<td><strong>Quad H kicks</strong></td>
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<tr>
<td>(\theta/\delta x_F) &amp; (\frac{K_F^{3/2}}{2}\tan(LK_F^{3/2}/2))</td>
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<tr>
<td><strong>Vertical c.o.</strong></td>
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<td><strong>Dipole V kicks</strong></td>
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<tr>
<td>(\theta/\delta z) &amp; (\tan(\alpha/\rho))</td>
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<tr>
<td>(\theta/\phi_x) &amp; (\sin(\theta/2)/\theta/2 - \cos(\theta/2 - \alpha)\cos(\alpha))</td>
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<tr>
<td>(\theta/\phi_z) &amp; (\sin(\theta/2))</td>
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<tr>
<td><strong>Quad V kicks</strong></td>
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<tr>
<td>(\theta/\delta z_F) &amp; (-\frac{K_F^{3/2}}{2}\tan(LK_F^{3/2}/2))</td>
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\(^{(a)}\) ± and \(\pm\) stand for entrance/exit kick signs, otherwise identical. 
\(^{(b)}\) Calculated for extreme values \(K=0.1 \text{ m}^{-2}\) and length=1 m.

### REFERENCES


