Experimental Studies of Resonance Crossing with a Paul Trap

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Use a Paul Trap?

- Paul trap found in Wikipedia.

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http://en.wikipedia.org/wiki/Main_Page
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Accelerator

Focusing

Defocusing

beam

Scheme of a Quadrupole ion trap of classical setup with a particle of positive charge (dark red), surrounded by a cloud of similarly charged particles (light red). The electric field $E$ (blue) is generated by a quadrupole of endcaps (a, positive) and a ring electrode (b). Picture 1 and 2 show two states during an AC cycle.

This is exactly an AG focusing system!

http://en.wikipedia.org/wiki/Main_Page
Outline

1. Introduction
   - Background and motivation
   - S-POD, an experimental tool for study of beam physics

2. Multi-particle Simulation
   - Resonance-band distribution in S-POD
   - Resonance crossing

3. Experiments
   - Resonance-band distribution in S-POD
   - Resonance crossing
   - Comparison with the multi-particle simulation
Background and Motivations

- Fixed-field accelerators have the potential to be a high-power accelerator owing to its high repetition rate.
- The beam optics potentially varies during the beam acceleration.
  - e.g. Non-Scaling FFAG (NS-FFAG) ring, EMMA
    Cell tune varies from ~0.3-4 to ~0.17
    The beam transverses one and more resonance bands.
- The resonance crossing may limit the machine performance.
- Past theoretical studies
  - Emitance growth is negligible or tolerable when the crossing speed is sufficiently high or/and the resonance is not so strong.

Today’s Talk

Experimental (and numerical) studies on betatron resonance crossing not using any accelerators, but using a plasma trap.
Study Beam Dynamics with a Plasma Trap

- Charged particle beam is...  
  a kind of non-neutral plasma confined in a machine.

- After some algebra, we reach following two Hamiltonians

\[
H_{\text{beam}} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(s)(x^2 - y^2) + \frac{q}{M\gamma^3(\beta c)^2} \phi
\]

\[
H_{\text{plasma}} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(\tau)(x^2 - y^2) + \frac{q}{M\gamma c^2} \phi
\]

with the Vlasov-Poisson equation

\[
\frac{\partial f}{\partial t} + [f, H] = 0 \quad \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi = -\frac{q}{\varepsilon_0} \iint f \, dp_x dp_y
\]

- We can use this physical equivalence to study beam dynamics.

Linear Paul trap

- Transverse confinement
  - RF quadrupole electric field
- Longitudinal confinement
  - Static potential barrier

Transverse 2D approximation \( \left( \frac{\partial}{\partial z} \approx 0 \right) \)

\[
H_{\text{plasma}} \approx \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(\tau) \left( x^2 - y^2 \right) + \frac{q}{Mc^2} \phi
\]
Advantages

- **Very compact and low cost**
  - Several tens of thousands dollars for the whole system

- **High flexibility of fundamental parameters**
  - Beam density, operating point, lattice function, etc.

- **High resolution & high precision measurements**
  - Faraday cup, micro-channel plate
  - Laser induced fluorescence (LIF)

- **Radio-activation free**
  - Experiment with any strong beam instability.
Linear Paul Trap System at Hiroshima Univ.

- S-POD - Simulator for Particle Orbit Dynamics –

- There are three S-POD systems
  - S-POD I: crystalline beam, nano-ion beam, etc.
  - S-POD II, S-POD III: resonant instability, etc.
Experimental Setup

- Multi-sectioned linear Paul trap

Ion species: $^{40}\text{Ar}^+$
Operating frequency: 1 MHz
RF amplitude: 0 ~ 92 V
Experimental Setup

- Multi-sectioned linear Paul trap

- Ion species: $^{40}\text{Ar}^+$
  - Operating frequency: 1 MHz
  - RF amplitude: 0 ~ 92 V

1. $^{40}\text{Ar}^+$ ions are generated by electron impact ionization with an electron.
Experimental Setup

- Multi-sectioned linear Paul trap

**Ion species**: $^{40}\text{Ar}^+$
- **Operating frequency**: 1 MHz
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**1.** $^{40}\text{Ar}^+$ ions are generated by electron impact ionization with an electron.

**2.** Storage plasma W/ or W/O tune excursion.

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### Experimental Setup

- **Multi-sectioned linear Paul trap**

![Diagram of a multi-sectioned linear Paul trap](image)

1. $^{40}$Ar$^+$ ions are generated by electron impact ionization with an electron.
2. Storage plasma W/ or W/O tune excursion.
3. Shut down the bias potential on Gate to send the plasma toward a FC detector.

**Graph:**
- **Ion species:** $^{40}$Ar$^+$
- **Operating frequency:** 1 MHz
- **RF amplitude:** 0 ~ 92 V

**Graph Details:**
- **Bare Tune $\nu_0$:**
- **RF Voltage [V]:**
- **Tune excursion by ramping the rf voltage**

**Other Details:**
- **Axial Potential Configuration**
- **End Cap A, IS, Gate, ER, End Cap B**
- **MCP, e-gun, Faraday Cup**

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Stop-band Distribution in S-POD

- **Experiment**
  - Tune survey on # of surviving particle after 10msec storage.

- **Simulation**
  - Emittance growth after 0.1msec

*Coherent resonance condition*

\[ \nu_0 - C_m \Delta \nu \approx \frac{n}{2m} \]

- The Instabilities at \( \nu_0 \sim 1/3 \) and 1/6 are likely enhanced by mechanical misalignment of electrodes in S-POD.

- Linear coherent resonance at \( \nu_0 \sim 1/4 \) is rapidly increased as the beam becomes denser.

Stop-band Distribution in S-POD

- **Experiment**
  - Tune survey on # of surviving particle after 10msec storage.

- **Simulation**
  - Emittance growth after 0.1msec

**Coherent resonance condition**

\[ \nu_0 - C_m \Delta \nu \approx \frac{n}{2m} \]

- \( m \): mode number
- \( n \): integer
- \( C_m \): constant \(< 1 \)

- The Instabilities at \( \nu_0 \sim 1/3 \) and \( 1/6 \) are likely enhanced by mechanical misalignment of electrodes in S-POD.

- Linear coherent resonance at \( \nu_0 \sim 1/4 \) is rapidly increased as the beam becomes denser.

EMMA
- Composed of 42 quadrupole-doublet cells along the ring.

S-POD
- 42 rf periods correspond to 1 turn around the EMMA lattice.
- Possible to study the effects of the lattice symmetry breaking by superimposing one or more lower-frequency rf waves.
Resonance Crossing – Low Density – PIC Simulation with the WARP code*

- Tune sweeping range (0.40 -> 0.17).
- 840 rf periods sweeping (= 20 turns along the EMMA ring)

\[ \frac{1}{n} \approx \frac{1}{4} \]

\[ \frac{1}{n} \approx \frac{1}{3} \]

Initial

\[ p_x \]

\[ x \]

Final

\[ \eta = 0.99 \]

* http://hifweb.lbl.gov/webpages/VNLsimulations.html
Resonance Crossing – High Density – PIC Simulation

- Tune sweeping range (0.40 -> 0.17)
- 840 rf periods sweeping (= 20 turns along the EMMA ring)

\[ \eta = 0.90 \]

Crossing of the quarter-integer tune is dangerous!

\[ \nu_0 \approx \frac{1}{4} \]

\[ \nu_0 \approx \frac{1}{3} \]
Faster crossing mitigates degradation of beam quality.

 Serious emittance growth is caused by the linear coherent resonance even with rather fast crossing.
Resonance Crossing Experiment

- Ions are ejected to the Faraday cup right after the tune sweeping.

- Low density cases (green & blue)
  - Ion losses are suppressed when the crossing speed is sufficiently high.

- High density case (red)
  - Ion Loss is not negligible even with rather high-speed crossing.

Crossing speed $u = \frac{\text{Crossing width}}{\text{Number of cell}} = \frac{\delta}{n_{rf}}$

Ion number vs. crossing speed

$N_{in} \sim 1 \times 10^7$

$N_{in} \sim 1 \times 10^6$

$N_{in} \sim 1 \times 10^5$

Tune excursion

$u \equiv \frac{\text{Crossing width}}{\text{Number of cell}} = \frac{\delta}{n_{rf}}$
Single Resonance Crossing

- Crossing of resonance at $v_0 \sim 1/6$
  - Negligible

- Crossing of resonance at $v_0 \sim 1/3$
  - Considerable ion losses
  - Enhanced by mechanical misalignment of the electrodes.
Single Resonance Crossing

- Linear coherent resonance at $\nu_0 \sim 1/4$
  
  - Ion loses are remarkably enhanced as particle number is increased.
  
  - This instability is instinct and independent of lattice errors.
  
  - Can be a troublesome issue for future high-density NS-FFAG beams.
Comparison with PIC Simulation on the Crossing of $\nu_0 \sim 1/4$ Resonance

- Well reproduce the experimental curve.
- Slower crossing or higher density beam results larger particle losses.
- Even with rather higher crossing speed, ion losses are not negligible in high-density beams.
- The performance of high-power NS-FFAGs maybe limited by this resonance.
Emittance Growth by the Crossing of $\nu_0 \sim 1/4$ Resonance - PIC Simulation -

- All distributions show qualitatively same behavior
- K-V beam can cross the resonance without large emittance growth as long as the tune depression and/or crossing speed is below the threshold value.

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S-POD Application to Integer Resonance Crossing
S.L. Sheehy et al., IPAC’13 2677.

- Crossing of multiple integer resonances in EMMA NS-FFAG.

- Dipole excitation

- Main focusing wave

- Dipole perturbation wave to emulate unexpected field from the injection septum.
Summary

- The S-POD system is employed to systematic study of betatron resonance crossing.
- Numerical campaign using the WARP PIC code is also conducted.
- As for low density beams, emittance dilution is negligible or tolerable when the crossing speed is sufficiently high or the resonance is not so strong.
- As for high density beams, linear coherent instability is dangerous even with rather high crossing speed.
- PIC simulation on emittance growth caused by the crossing of $\nu_0 \sim 1/4$ resonance.
  - Gaussian, K-V Semi-Gaussian and Waterbag beams have same tendency.
  - K-V beam is the most stable as long as the tune depression and crossing speed is below the threshold value.
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    Jean-Luc Vay
Thank you for your attention!
Appendix
Penning-Malmberg trap

- Transverse confinement
  - Axial magnetic field
- Longitudinal confinement
  - Static potential barrier

2D Hamiltonian in a rotating frame

\[ H_{\text{plasma}} \approx \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(x^2 + y^2) + \frac{q}{M c^2} \phi \]

Smooth focusing lattice

Study of beam halo formation

Choice of Lattice Function

- The S-POD system can generate a wide variety of lattice functions.

- FODO lattice and sinusoidal focusing system have almost an identical resonance structure.

- We employ the sinusoidal focusing just for technical simplicity.

Resonance Crossing – Distribution Dependency – PIC Simulation

Tune depression: $\eta = 0.90$
420 rf periods sweeping

- Qualitatively same emittance evolution.
- KV beam results the smallest emittance growth.
### Simulation Setup

- Transverse 2D simulation using PIC code WARP*
  - The 2D approximation is reasonable because…
  - Longitudinal potential wall is square-like.

- Launch a plasma matched to the focusing force and see how does it evolve.

### Simulation Parameters

Initial distribution:
- Gaussian, KV, Waterbag, Semi-Gaussian

Temperature: 0.1 ~ 0.3 eV

Tune depression $\eta$: 0.8 ~ 1.0

# of simulation particles: $10^5$

# of integration step: 200 per 1 rf period

* [http://hifweb.lbl.gov/webpages/VNLsimulations.html](http://hifweb.lbl.gov/webpages/VNLsimulations.html)