Abstract

In ring experiments at the heavy ion storage ring, using a reaction microscope, require highly charged bunched ion beams with bunch length below 5 ns. Small longitudinal ion profiles can be obtained by bunching the ion beam with electron cooling. The measured short bunch lengths are determined by the space charge limit. To over come the space charge limit and to further minimize the bunch length, the TSR was operated at a momentum compaction factor 1.58, a mode in which the revolution frequency at higher energies decreased. This reduced the bunch length by up to 3.5 times compared to the standard mode. During this beam time, self-bunching of the ion beam was observed for the first time in the TSR. To provide highly charge ions at the TSR deceleration is required. Deceleration experiments are mainly carried out with $^{12}$C$^{6+}$ ions to investigate the behavior and evolution of the beam during deceleration. To explore the deceleration cycle, $^{12}$C$^{6+}$ ions are decelerated from 73.3 MeV to 9.77 MeV with an efficiency of about 90 %. To achieve this low energy two cooling steps at the initial and final bunch energies are applied. Electron pre-cooling results in a dense ion beam where IBS has to be taken into account to describe the development of the beam size during deceleration. An approximated model of IBS is proposed to interpret the experimental data.

SHORT ION BUNCHES

Small longitudinal bunch lengths are necessary for experiments with a reaction microscope in a storage ring. Tests therefore were performed with 50 MeV $^{12}$C$^{6+}$ ion beams using the 6th harmonic for bunching. A bunched ion beam profile obtained with simultaneous electron cooling, measured with a capacitive pick-up, is shown in Figure 1. The intensity of the $^{12}$C$^{6+}$ ion beam with $E = 50$ MeV used for this measurements was $I = 45 \, \mu A$. The resonator voltage was set to 795 V. Also shown in Fig. 1 is a parabola fit function (red line), which represents the data very well. A bunch length, defined in Fig. 1, of $w = 20 \, \text{ns}$ can be obtained from the fit. This bunch length is space charge limited. In the space charge limit the voltage of the resonator $U_i(\Delta \phi) = U \sin(\Delta \phi + \phi_s)$ each ion is passing through is compensated by the longitudinal space charge voltage of the ion beam. For bunching, in the TSR standard mode, where the slip factor $\eta = \frac{f_0/p}{2\pi f_i}$ is positive, the synchronous phase used for bunching is $\phi_s = 0$, where $f_0$ is the revolution frequency of an ion and $p$ describes its momentum. Because the synchrotron oscillation is a very slow process compared to the revolution time, the longitudinal electrical field $E_{||}(\Delta \phi)$, seen by one ion, can be assumed to be constant during one turn and the space charge voltage can be defined by $U_i(\Delta \phi) = E_{||}(\Delta \phi) \cdot C_0$, where $C_0$ denotes the circumference of the storage ring. The ion phase $\Delta \phi$ is related to the longitudinal position $s$ in the bunch: $\Delta \phi = -\frac{w}{2 \gamma}$, where $\omega$ is the angular frequency of the resonator and $v_0$ the velocity of the synchronous particle, located in the center of the bunch at $s=0$. Ions in front of the synchronous particle ($s > 0$) arrive at the resonator gap earlier than the synchronous one, therefore there is a negative sign in the formula. The longitudinal electrical field $E_{||}(s)$ can be calculated from the charge line density $\lambda(s)$ of the bunch by the following formula [1]:

$$E_{||}(s) = -\frac{1 + 2 \ln \left( \frac{R}{r} \right)}{4 \pi \varepsilon_0 \gamma^2} \frac{\partial \lambda(s)}{\partial s}. \tag{1}$$

The constant $\varepsilon_0$ is the absolute permittivity and $\gamma$ is the relativistic mass increase (for TSR energies $\gamma = 1$). $R$ denotes the radius of the beam tube ($R = 0.1 \, \text{m}$) and $r$ is the average beam radius, defined by twice the two $\sigma_r$ value ($r = 2 \sigma_r$) of the transverse beam width. A parabola density profile is the only longitudinal charge line distribution, for an electron cooled ion beam with $\Delta \phi \ll 2 \pi \left( \sin(\Delta \phi) = \Delta \phi \right)$, which compensates the resonator voltage $U_i(\Delta \phi)$ for each ion, independent of its phase $\Delta \phi$. The parabola charge line density $\lambda(s)$ can be calculated from the number $N_B$ of particle in the bunch:

$$\lambda(s) = \frac{3 N_B Q}{4 w_s} \left( 1 - \frac{s^2}{w_s^2} \right) \tag{2}$$

for $|s| \leq w_s$, with $\int_{-w_s}^{w_s} \lambda(s) \, ds = N_B \cdot Q$. The charge of an ion is $Q$ and $w_s$ describes the bunch length in meters, re-

Figure 1: Measured electron cooled longitudinal ion beam ($^{12}$C$^{6+}$, $E = 50 \, \text{MeV}$) profile. The width of the parabola profile is defined by $w$. 

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lated to the bunch length $w$ in seconds, $w = v_s \cdot w$, defined in Fig. 1. If $U_i(\Delta \phi)$ is completely compensated by the space charge voltage $U \cdot \sin(\Delta \phi + \phi_s) + E_i(\Delta \phi) \cdot C_0 = 0$, the synchrotron oscillation of each particle in the bunch is freezeed. This condition leads finally to the longitudinal space charge limit. For a beam, having a parabola longitudinal charge line density, the space charge limit is given by following formula:

$$w = C_0 \sqrt{\frac{3(1 + 2 \ln(\frac{R}{r}))I}{2 \pi^2 c^4 e_0 \gamma^2 h^2 \beta^4 U}}.$$  

(3)

The bunch length $w$ in formula (3) is determined by the beam intensity $I$, the resonator voltage $U$, the number of bunches $h$ in the ring and the beam velocity $\beta$ in units of the speed of light $c$. If the space charge voltage $|U_s(\Delta \phi)|$ of the ion beam would be larger than $|U_i(\Delta \phi)|$, the magnitude $|\Delta \phi|$ of each ion would increase by the repelling space charge force, resulting in an increase of the bunch length. On the other hand a larger bunch has a smaller space charge voltage $|U_s(\Delta \phi)|$, thus the ion starts to oscillate. These oscillations $\Delta \phi$ will be damped by the electron cooler, bringing back the beam to the space charge limit. Therefore an electron cooled ion bunch in the space charge limit is stable. With an average transverse beam seam size $\sigma_r=1 \text{ mm}$, the bunch length $w$ can be calculated. Figure 2 shows the measured bunch length $w$ as a function of resonator voltage $U$ as well the theoretical prediction (red curve). As it is shown in Fig. 2 the calculated function with the fit to the data ($w \sim U^{-0.34}$), blue line, agrees very well. At the same number of bunches $h = 6$, the bunch length $w$ was measured as a function of the beam intensity, shown in Fig. 3. The resonator voltage used in these measurements was $U = 795 \text{ V}$. A fit through the data, blue curve, gives an exponent of 0.31, which is slightly less than the predicted value of 1/3.

**Figure 2**: Measured bunch length $w$ for an electron cooled $^{12}\text{C}^{6+}$ ion beam ($E = 50 \text{ MeV}$, $I = 20 \mu \text{A}$) as a function of the resonator voltage [2]. The red curve is a calculation, where formula (3) was used.

**Figure 3**: Measured bunch length for an electron cooled $^{12}\text{C}^{6+}$ ion beam as a function of the ion intensity [2]. The resonator voltage used in this measurement was $U = 795 \text{ V}$. The red curve is a calculation using formula (3).

Bunch Lengths at Negative $\eta$

The ion bunch length can be decreased by increasing the resonator voltage $U$ or by decreasing the intensity $I$ of the stored ion beam. But for both quantities there are practical limits. The intensity limit is given by the experimental requirements, whereas the voltage is limited by the maximum voltage of the resonator, which should not exceed in our case 5 kV. To decrease the bunch length further the space charge limit has to be overcome. Because the synchrotron frequency $f_{sy}$ fulfills the following relation: $f_{sy} \sim \sqrt{\cos(\phi_s)}$, bunching is done at $\phi_s = \pi$ for $\eta = 2f_{sy}/f_0 < 0$, to obtain a real synchrotron frequency. If the beam is bunched at $\phi_s = \pi$, the voltage $U_i(\Delta \phi)$, seen by one ion, has the same sign as the space charge voltage, thus the space charge of the ion beam $U_s(\Delta \phi)$ cannot compensate $U_i(\Delta \phi)$. A negative $\eta$ parameter means that particles with larger momentum than the central particle need more times $T$ ($T = 1/f_0$) for one turn compared to the central one. A negative slip factor $\eta$ can be achieved by increasing the length of the closed orbit for an ion having a positive momentum deviation. The length of the closed orbit $C_0$ can be described by the momentum compaction $\alpha = \frac{\Delta C_0/C_0}{\Delta p/p_0}$ of the storage ring. To avoid the space charge effect, the TSR was set to $\alpha=1.58$, which is consistent with $\eta = -0.59$, for 50 MeV $^{12}\text{C}^{6+}$ ions. An $\alpha$ parameter of 1.58 results in an average dispersion $\bar{D}_x = 14 \text{ m}$ in the TSR main dipole magnets. At this setting bunch length measurements for an electron cooled 50 MeV $^{12}\text{C}^{6+}$ ion beam were performed. A longitudinal ion bunch profile taken at a beam intensity of $I = 2.9 \mu \text{A}$ and at a rf frequency $f = 3.0504 \text{ MHz}$ is shown in Fig. 4. In contrast to the profile measured at the space charge limit, this profile can be described with a Gaussian distribution. At the intensity of $I = 2.9 \mu \text{A}$ a beam width $\sigma=3.03 \text{ ns}$ was determined. The measured beam widths $\sigma$ for different resonator voltages and intensities in comparison with the beam width
measured in the standard mode are shown in Table 1. To compare this Gaussian bunch length $\sigma$ with the parabola bunch length $w$ obtained at the space charge limit, the parabola bunch length has to be converted to a corresponding length $\sigma^*$ of a Gaussian distribution, having the same half width. A parabola and a Gaussian distribution have the same half width if the relation: $\sigma^* = \frac{w}{2\sqrt{\ln(2)}}$ is fulfilled.

For 50 MeV $^{12}C^{6+}$ ions the space charge limit is given by: $w[\text{ns}] = 62.1 \frac{I[\text{mA}]}{U[\text{kV}]}$. With this equation the corresponding Gaussian bunch lengths $\sigma^*$ for the resonator voltages and intensities shown in Table 1 can be calculated. As indicated in the table the bunch lengths at negative $\eta$ values are about factor three smaller than the corresponding bunch length in the standard mode. The measured bunch length of $\sigma \approx 3$ ns corresponds to a spatial length of 8.5 cm, which is nearly twice of the pick-up length used to measure the bunch shape. Therefore the actual bunch lengths measured at negative $\eta$ parameter should be somewhat below the values $\sigma_{\eta<0}$ shown in Table 1.

**Self-Bunching of an Electron Cooled 50 MeV $^{12}C^{6+}$ Beam**

At negative $\eta$ values self bunching of the ion beam can occur if a certain threshold current $I_{th}$ ($I_{th} \sim \Delta p^2$) [3], depending on the momentum spread $\Delta p$ of the ion beam is exceeded. This effect, called negative mass instability [3, 4], was indeed observed at an electron cooled 50 MeV $^{12}C^{6+}$ ion beam. The voltage signal recorded on a capacitive pick-up at an ion Intensity $I \approx 2$ mA, without bunching the beam fit leading to a bunch length of $\sigma = 4.5$ ns. Furthermore it was investigated how long it takes after injection to develop this bunch structure. For that purpose a spectrum analyzer in span zero mode was used to measure the time development of the bunch spectrum at $f \approx 1$ MHz, corresponding to a harmonic number of two. The result of this measurement is displayed in Figure 7. At the time $t=0$ s the carbon ion beam was injected into the TSR and electron cooling starts immediately. As it is shown in Figure 7 self bunching starts at about 150 ms after injection and at $t = 0.4 - 0.5$ s the bunches are formed due to self bunching. After $t \approx 0.5$ s there is a decay of the amplitude of the bunch spectrum with a decay time of $\tau_d \approx 6s$ visible, measured at $f \approx 1$ MHz. The decay time $\tau_d$ is much shorter than the lifetime ($\tau \approx 1400$ s) of the electron cooled ion
beam.

**INTRA-BEAM SCATTERING EFFECTS DURING DECELERATION**

Atomic collision experiments sometimes require intense beams of multi-charged ions at medium and low energies. The production of the low energy ions is accomplished at the TSR by deceleration. To decelerate a heavy ion beam, electron cooling at the injection energy is required, resulting in a dense ion beam where intra-beam scattering effects has to be taken into account. Just before starting the deceleration cycle electron cooling is switched off and the ion beam size increases due to intra beam scattering effects. The blow up rates of bunched beams due to intra beam scattering (IBS) can be expressed by [5, 6]:

$$\frac{1}{\sigma_i} \frac{d\sigma_i}{dt} = \frac{q^4 N}{A^2 \beta^3 \epsilon_x \epsilon_y \Delta p / p \cdot h \cdot l_{eff}},$$  \hspace{1cm} (4)

where $i (i=x, y, \Delta p)$ denotes the horizontal, vertical and longitudinal coordinates of the beam. $N$ describes the number of ions which charge state $q$, mass $A$ and velocity $\beta$. The number of bunches in the storage ring is $h$ and $l_{eff}$ is the effective bunch length. $c_i$ are lattice dependent functions with are weakly depends on the ion energy. The horizontal emittance as well the vertical emittance scales as $\epsilon_x \propto \sigma_x^2$, $\epsilon_y \propto \sigma_y^2$, where $\sigma_x$ and $\sigma_y$ are the horizontal and vertical beam width and $l_{eff}$ is the effective bunch length.

If in the IBS process, starting from the equilibrium between IBS and electron cooling, the horizontal beam width $\sigma_x$, the vertical beam width $\sigma_y$, the momentum spread $\Delta p / p$ and the effective bunch length are proportional to each other: $\epsilon_x \epsilon_y l_{eff} \Delta p / p \propto \sigma_i^2$ we obtain three uncoupled differential equation for all three degree of freedom [7, 8]:

$$\frac{1}{\sigma_i} \frac{d\sigma_i}{dt} = \frac{D_i}{\sigma_i^3},$$ \hspace{1cm} (5)

In our simply model $\gamma = 6$ [8] for a bunched ion beam and

$$D_i \propto c_i \frac{q^4}{A^2 \beta^3} \frac{N}{h},$$ \hspace{1cm} (6)

If the velocity of the ion beam is not changed the solution of Equation (5) is given by:

$$\sigma_i(t) = (\sigma_{i,0} + \gamma D_i t)^{\frac{1}{\gamma}},$$ \hspace{1cm} (7)

where $\sigma_{i,0}$ is the initial beam width. In Figure 8 the measured horizontal beam width of a $^{12}$C$^{6+}$ bunched ion beam at the injection energy $E=73.3$ MeV is shown as a function of time. The dashed curve through the data is a fit where $\gamma$ is used as an fit parameter. For bunching a resonator voltage of 186 V was used. From the fit we obtained: $\gamma = 5.9$, which is close to the theoretical one of $\gamma = 6$. As it is shown in Figure 8 the horizontal profile can be described very well with our simply IBS model if the ion beam velocity is keep constant. To investigate the velocity dependence of IBS we make following ansatz for the heating term:

$$D_i = \frac{\dot{D}_i}{\beta^2},$$ \hspace{1cm} (8)

where $\kappa$ should be close to 3. In the deceleration cycle the ion velocity is changed linear:

$$\beta(t) = \beta_0 + \alpha t,$$ \hspace{1cm} (9)

where $\beta_0$ is the initial velocity. The parameter $\alpha$ can be calculated from the initial $\beta_0$, final ion velocity $\beta_f$ and ramping time $T$: $\alpha = (\beta_f - \beta_0) / T$. To calculate the beam width during deceleration we have to solve Equation (5) with (8) and (9):

$$\sigma_i(t) = (\sigma_{i,0} + \gamma \dot{D}_i (\beta_0^{1-\kappa} - (\beta_0 + \alpha t)^{1-\kappa}))^{\frac{1}{\gamma}}$$ \hspace{1cm} (10)

From Equation (10) it follows that the beam width for very weak intensities ($D_i \to 0$) is constant during deceleration. But this is not the case. Deceleration of very weak beams can be described by the theorem of Liouville, postulating that the phase space area of noninteracting particle is conserved during deceleration. The Liouville theorem leads to an increase of the ion beam size during deceleration. In the

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**Figure 7:** Measurement of the pick-up signal at the second harmonic number ($f \approx 1$ MHz) of the revolution frequency as a function of time.

**Figure 8:** Measured horizontal beam width $\sigma$ of a $^{12}$C$^{6+}$ bunched ion beam ($E=73.3$ MeV) as a function of time after switching of the electron cooler. The intensity used in that measurement was $I = 50\mu A$. 

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non relativistic approach the change of the ion beam size \( \sigma_i \) is given:

\[
\sigma_i(t) = \tilde{\sigma}_i \sqrt{\frac{\beta_0}{\beta(t)}},
\]  

(11)

where \( \beta(t) \) is the ion velocity at the time \( t \) and \( \tilde{\sigma}_i \) is the beam with at \( t=0 \) s. Equation (11) is the boundary value for \( D_i \rightarrow 0 \) of the equation \( \sigma_i(t) \), which we are looking for. To take intra-beam scattering into account we replace \( \tilde{\sigma}_i \) in Equation (11) with the beam width \( \sigma_i(t) \) in Equation (10) [6]:

\[
\sigma_i(t) = \left( \frac{\gamma \tilde{D}_i}{\sigma_{x,0}} \left( \frac{\beta_0^{1-\kappa} - \beta(t)^{1-\kappa}}{\alpha^{\kappa-1}} \right) \right)^{\frac{1}{\kappa}} \sqrt{\frac{\beta_0}{\beta(t)}},
\]  

(12)

where \( \beta(t) = \beta_0 + \alpha t \). With Equation (12) the beam width during deceleration can be calculated. In Figure 9 the measured horizontal beam width during deceleration as a function of time is shown. The injected intensity at this experiment was \( I = 28 \mu A \). After injection the ion beam was electron cooled, resulting in a dense horizontal beam width \( \sigma_x = 0.05 \) mm. At the time \( t = 4 \) s electron cooling was switched off and the ion beam was decelerated in \( 7 \) s to the final energy of \( 9.7 \) MeV. After reaching the final energy at \( t = 11 \) s electron cooling was switched on again, noticeable in the very fast reduction of the horizontal beam width \( \sigma_x \). Because the intra beam scattering heating term \( D_i \) compare (Equation 6 ) is much higher at the final energy, the equilibrium beam width at the final energy of \( E=9.7 \) MeV is larger compared to the electron cooled beam width at the initial energy of \( E=73.3 \) MeV. The calculation of the beam width, using Equation 12, are shown in Figure 9 as a red line. The IBS heating term \( \tilde{D}_i \) and \( \gamma \) were determined at the injection energy \( E=73.3 \) MeV [8]. The equilibrium beam width \( \sigma_{x,0} \) was calculated from the measured beam widths between \( t=2-4 \) s. For \( \kappa \) the theoretical value of \( \kappa = 3 \) was used in the evaluation of the red line. As it is pointed out in Figure 9 the calculation of the beam width during deceleration using only IBS data, measured at the injection energy, is in a good agreement with the observation.

**CONCLUSION**

For a bunched electron cooled ion beam \( ^{12}C^{6+}, E = 50 \) MeV), a bunch length of \( w = 3.1 \) ns at \( h = 6 \) and \( I=0.1 \mu A \) can be anticipated in the TSR standard mode (\( \eta > 0 \)), sufficient for experiments with an internal gas jet target and a reaction microscope. To overcome the space charge limit the TSR was operated at a momentum compaction of \( \alpha = 1.58 \). In this mode much shorter bunch lengths (about factor 2-3.5) compared to the standard mode were achieved, if the same intensity and resonator voltage \( U \) are used. In experiments devoted to deceleration of \( ^{12}C^{6+} \) ions, a reduction of the beam energy by a factor of \( > 7 \), from \( 73.3 \) MeV to \( 9.7 \) MeV (1 MeV/u), corresponding to a rigidity decrease from \( 0.71 \) Tm to \( 0.26 \) Tm, could be achieved with an efficiency of about 90% [6]. In order to obtain this high efficiency electron pre-cooling is required, resulting in a dense ion beam where IBS effects has to be taken into account to describe the development of the ion beam width during deceleration. Equation (12) which we derived to describe the beam width in the deceleration process is in good agreement with the observation.

**REFERENCES**